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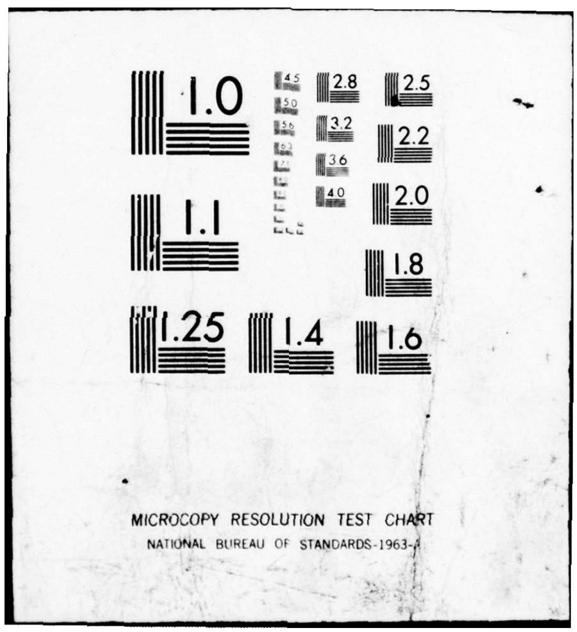
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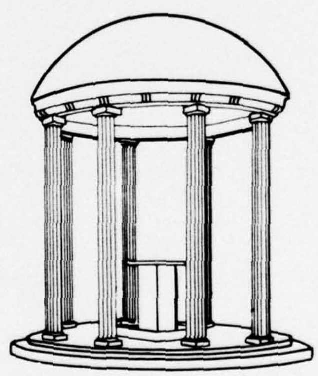



## SCHOOL OF BUSINESS ADMINISTRATION
## AND
## CURRICULUM IN
## OPERATIONS RESEARCH AND SYSTEMS ANALYSIS

# FORECASTING OPERATING CHARACTERISTICS OF (s,S) INVENTORY SYSTEMS

Technical Report #13

Carl R. Schultz, Richard Ehrhardt
and Alastair MacCormick*

December 1977



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The report ~~We~~ consider the management of an inventory control system operating under incomplete information about the probability distribution of demand. The system control policy is of an (s,S) form where the policy parameters, s and S, are periodically revised using a demand history of fixed length. In such a system it is often desirable to forecast values of such key system operating characteristics as: total cost per period, period-end

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inventory, period-end backlog quantity, backlog frequency, and replenishment frequency.

Several methods of forecasting system performance are presented. The criterion used to evaluate and compare these methods is the extent of prediction bias and its level of dispersion. Most of these procedures require estimation of the underlying demand distribution from a limited sample of historical data. In ~~We have found for our~~ 16-item system, ~~that~~ forecasts of aggregate total cost are typically 20% below actual values. Much of this error results from underestimation of backlog quantities. This is due to the fact that the demand data is used both to set the policy parameters and to forecast system behavior.

MacCormick, A. (1975), Statistical Problems in Inventory Control, ONR and ARO Technical Report, December 1974, School of Organization and Management, Yale University, 244 pp.

Estey, A. S. and R. L. Kaufman (1975), Multi Item Inventory System Policies Using Statistical Estimates: Negative Binomial Demands (Variance/Mean = 1), ONR and ARO Technical Report 3, September 1975, School of Organization and Management, Yale University, 85 pp.

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Kaufman, R. and J. Klincewicz (1976), Multi-Item Inventory System Policies Using Statistical Estimates: Sporadic Demands (Variance/Mean = 9), ONR and ARO Technical Report 6, School of Organization and Management, Yale University, 58 pp.

Ehrhardt, R. (1976), The Power Approximation: Inventory Policies Based on Limited Demand Information, ONR and ARO Technical Report 7, School of Organization and Management, Yale University, 58 pp.

Klincewicz, J. G. (1976), Biased Variance Estimators for Statistical Inventory Policies, ONR and ARO Technical Report 8, School of Organization and Management, Yale University, 24 pp.

Klincewicz, J. G. (1976), Inventory Control Using Statistical Estimates: The Power Approximation and Sporadic Demands (Variance/Mean = 9), ONR and ARO Technical Report 9, School of Organization and Management, Yale University, 52 pp.

Klincewicz, J.G. (1976), The Power Approximation: Control of Multi-item Inventory Systems with Constant Standard-deviation-to-mean Ratio for Demand, ONR and ARO Technical Report 10, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 47 pp.

Kaufman, R. L. (1977), (s,S) Inventory Policies in a Nonstationary Demand Environment, ARO Technical Report 11, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 155 pp.

Ehrhardt, R. (1977), Operating Characteristic Approximations for the Analysis of (s,S) Inventory Systems, ONR and ARO Technical Report 12, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 109 pp.

## FOREWORD

As part of the on-going research program in "Decision Control Models in Operations Research", the authors of this report have investigated several methods of forecasting the performance of statistically-controlled inventory systems. This work is an attempt to improve upon the bias and variability of the method of retrospective simulation which was investigated in previous studies (see Technical Report Nos. 2, 3, and 7). Although the authors report only partial success, several new avenues of research have emerged. These will be discussed in subsequent Technical Reports.

A list of other related reports dealing with this research program follows.

Harvey M. Wagner
Principal Investigator

Richard Ehrhardt
Co-principal Investigator

CONTENTS

## ABSTRACT

We consider the management of an inventory control system operating under incomplete information about the probability distribution of demand. The system control policy is of an (s,S) form where the policy parameters, s and S, are periodically revised using a demand history of fixed length. In such a system it is often desirable to forecast values of such key system operating characteristics as: total cost per period, period-end inventory, period-end backlog quantity, backlog frequency, and replenishment frequency.

Several methods of forecasting system performance are presented. The criterion used to evaluate and compare these methods is the extent of prediction bias and its level of dispersion. Most of these procedures require estimation of the underlying demand distribution from a limited sample of historical data. We have found for our 16-item system, that forecasts of aggregate total cost are typically 20% below actual values. Much of this error results from underestimation of backlog quantities. This is due to the fact that the demand data is used both to set the policy parameters and to forecast system behavior.

## 1. OPERATING CHARACTERISTICS OF (s,S) INVENTORY SYSTEMS

Consider an inventory manager who must design a system of replenishment rules for the stockage of possibly thousands of items. Assume that the manager knows the costs of holding inventory, making replenishment decisions, revising the stock status of an item, etc., and is able to attach a cost to each unit of demand that cannot be filled. Provided that the relevant operating characteristics for each system can be obtained, and that he has a preference ordering, the inventory manager is able to determine whether one system is better than another.

In designing the system, the manager must select a class of decision rules, and preferably pick optimal policies from the selected class. For example, the complete economic preference function of the manager may imply that if the probability distribution of demand is known, then the optimal policy is of an (s,S) form: when inventory on hand plus on order falls below s, place an order so that inventory on hand plus on order is S. However, it will often be the case that the manager faces considerable uncertainty about the underlying demand distribution. Consequently, a decision process for systems design becomes much more complex than the mere selection of values for (s,S).

First, if the manager suspects a non-stationary demand environment, he must decide how much past demand data to actually use in updating the replenishment policy parameters and how often these revisions should be made. Second, the manager must either guess at the form of the demand distribution, statistically estimate the distribution's parameters, and then compute optimal policies for that distribution, or he can choose an

"approximately optimal" form for the replenishment policy that depends on only a few demand parameters, such as the mean and variance. Third, the manager must decide, before implementing the system, whether the design parameters are well set. To accomplish this task, the manager will need to predict the system's performance by forecasting the values of key operating characteristics. Usually, the manager will employ the same demands for forecasting as well as for policy derivations. This double use of the demand data, however, leads to biased forecasts. To eliminate this bias, the manager might consider splitting the demand sample in half; using one half of the sample to update the policy parameters, and the other half to forecast the policy's performance. However, the manager will still want to use a policy based on the entire demand sample. Thus, before this method can be investigated, we need an understanding of the relationship between the split-sample forecast and the performance of the policy based on the entire demand sample. This procedure, along with several other forecasting methods currently under investigation, is mentioned as a topic for future research in the final section of this report.

It is the purpose of this study to examine several alternatives for forecasting inventory system operating characteristics. The methods we consider fall into two major categories: methods that use approximate formulae for the operating characteristics, examined in Section 2, and methods that rely on simulation to estimate the characteristics, examined in Section 3.

## 1.1 The Model

Throughout this report, we deal with a single-item inventory model. It is assumed that inventory levels are reviewed periodically, and that the demand for an item is described by a stationary, discrete-time, stochastic process. The demand sequence, denoted $\xi_1, \xi_2, \ldots,$ consists of independent, identically distributed, non-negative, integer valued random variables.

As long as the amount of stock on hand is sufficient, demands are met; should a stockout occur, however, the unsatisfied demand is completely backlogged until sufficient replenishments arrive.

Items kept in inventory are assumed to be conserved, there being no losses by deterioration, obsolesence, or pilferage; disposal is not allowed. Inventory on hand at the end of a period is the inventory from the previous period plus any replenishment that arrives, less any demand. Negative stock-on-hand represents the amount backlogged. Replenishments are assumed to be delivered after a known fixed leadtime $L$ periods after being ordered. The time sequence of events in any period is taken to be order, delivery, demand.

We assume no time discounting of costs and an unbounded horizon over which an item is demanded and stocked. Our objective is to minimize expected total cost per period.

The cost of a replenishment quantity $q$, denoted $c(q)$, is assumed to be of the form

$$c(q) = \begin{cases} k + cq & \text{For } q > 0 \\ 0 & \text{For } q = 0 \end{cases}$$

where $k$ is the fixed ordering cost and $c$ is the constant cost per item. Since costs are not discounted and unfilled demands are completely backlogged, the constant unit cost $c$ is not a factor in choosing a minimal cost policy, and is hereafter suppressed.

Let $i$ represent the inventory-on-hand at the end of any period. If $i > 0$, we assume that a holding cost proportional to $i$ at unit cost $h$ is assessed. If $i < 0$, we assume a backlog penalty cost proportional to $-i$ at unit cost $p$ is charged.

The resulting total cost function is linear in $k$, $p$ and $h$, and we can scale these parameters so that the value of the unit holding cost $h$ is one. Non-trivial changes in costs arise only with changes in the ratios $k/h$ and $p/h$.

## 1.2 The Inventory Replenishment Policy

We postulate that control over replenishment in the inventory system is exercised by an $(s,S)$ policy: whenever inventory $x$ on hand and on order at the start of a period drops below the value $s$, an order is placed for a replenishment of size $S - x$. Given our assumptions, when the demand distribution and the economic parameters are known, there is an optimal policy that has the $(s,S)$ form [Iglehart (1963a,b), Veinott and Wagner (1965)]. When the demand distribution is not known, even though this is the only assumption relaxed, an optimal policy may no longer be of the $(s,S)$ form. Nevertheless, in this study we use an $(s,S)$ policy since it is in popular use in the applied situation of incomplete information.

For the purposes of this study we compute approximately optimal $(s,S)$

policy rules by way of the Statistical Power Approximation [Ehrhardt (1976)].

The Power Approximation is an algorithm for computing approximately optimal values for (s,S) using only the mean $\mu$ and variance $\sigma^2$ of demand. The algorithm is executed as follows. Let

$$(1) \quad D_p = (1.463)\mu^{.364}(k/h)^{.498}[(L+1)\sigma^2]^{.0691}$$

and

$$(2) \quad \begin{aligned} s_1 &= (L+1)\mu + [(L+1)\mu]^{.416}(\sigma^2/\mu)^{.603}U(z) \\ S_1 &= s_1 + D_p \end{aligned}$$

where $U(z)$ is given by

$$U(z) = .182/z + 1.142 - 3.466z \, ,$$

$$(3) \quad z = \left\{ \frac{\mu^{.364}(k/h)^{.498}}{(1+p/h)[(L+1)\sigma^2]^{.431}} \right\}^{1/2}$$

If $D_p/\mu$ is greater than 1.5, let $s = s_1$ and $S = S_1$. Otherwise, compute

$$(4) \quad S_2 = (L+1)\mu + v[(L+1)\sigma^2]^{1/2} \, ,$$

where $v$ is the solution to

$$(5) \quad \int_{-\infty}^{v} (2\pi)^{-1/2} \exp(-x^2/2)dx = p/(p + h) .$$

The policy parameters are then given by

$$(6) \quad \begin{aligned} s &= \min\{s_1, S_2\} \\ S &= \min\{S_1, S_2\} \end{aligned} .$$

When demands are integer valued, $s_1$, $D_p$, and $S_2$ are rounded to the nearest integer.

The Power Approximation assumes that the mean and variance of demand are known. For the more realistic situation in which only sample statistics of previous demands are available to the decision maker, the policy parameters, $s$ and $S$, can be obtained by the Statistical Power Approximation, which is simply the Power Approximation with sample estimates for the mean and variance of demand substituted for $\mu$ and $\sigma^2$ in equations (1) through (6).

If the underlying demand sequence is stationary the entire demand history should be accumulated to yield progressively better performance; however, the inventory manager is not in a position to know that the conditions observed to date will continue to prevail. For this reason the policy parameters are periodically revised.

For our study the admittedly arbitrary choice has been made to keep a history of fixed length and to give equal weight to all observations in this

history. Let $T$ be the length of the revision interval; that is, the number of periods between revisions. Assume that a history of $T$ periods' demand is kept for use at each revision. If $t$ is a period at the beginning of which a revision is made, then

$$\bar{\xi} = T^{-1} \sum_{\tau=1}^{T} \xi_{t-\tau}$$

(7) and

$$\bar{v} = (T - 1)^{-1} \sum_{\tau=1}^{T} (\xi_{t-\tau} - \bar{\xi})^2$$

are, respectively, the sample estimates for the demand mean $\mu$ and variance $\sigma^2$ required by our decision rules. Thus when using the Statistical Power Approximation we periodically obtain values for (s,S) by substituting $\bar{\xi}$ and $\bar{v}$ for $\mu$ and $\sigma^2$ in equations (1) through (6).

1.3 Forecasting the Performance of Inventory Systems Under Statistical Control Policies

In our study we are interested in predicting system performance by examining forecasts of key operating characteristics. These system operating characteristics are: period-end holding quantity, period-end backlog quantity, frequency of period-end backlogs, frequency of replenishment, and total cost.

Consider a system in which the control parameters are revised periodically. At the beginning of each revision interval the control parameters (s,S) are revised and a forecast $f$ is made of the mean value of each operating characteristic over the revision interval. Let $a$ be the actual realization

of the mean operating characteristic. If the process continues over an infinite horizon, statistical equilibrium will be established and the random variables $f$ and $a$ will be identically distributed, though correlated, from one period to the next.

We characterize the quality of the forecasting method by its bias $B$, defined as

$$(8) \quad B = E(a - f)$$

and by its level of dispersion $D$, where

$$(9) \quad D = [Var(a - f)]^{1/2} \; .$$

Note that our definition of bias is the negative of the definition usually given for bias, since our biases run in this direction.

In our study we estimate $B$ and $D$ for a given forecasting method from the output of a computer simulation. Let $f_i$ and $a_i$ be, respectively, the forecast and actual operating characteristics in revision interval $i$ of the simulation experiment. Simulated data are collected for 200 consecutive revision intervals. The sample mean $m$ and sample variance $v$ of the quantity $(a_i - f_i)$ are constructed using an autoregressive representation of the time series $\{(a_i - f_i), i = 1,..., 200\}$. We construct estimates $b$ and $d$ of the quantities $B$ and $D$, respectively, where

$$b = m,$$

and

$$d = \sqrt{v}$$

Our objective is to obtain a forecasting procedure which produces forecasts with biases which are close to zero and whose level of dispersion is low.

## 1.4 Forecasts by Retrospective Simulation

Previous forecasting studies [MacCormick (1974), Estey and Kaufman (1975), and Ehrhardt (1976)] have utilized a method called "retrospective simulation". This method entails using the same sample of demands both to set the policy parameters (s,S), and to forecast the system's performance by simulating how the policy would have performed on these demands. For a complete description of this procedure see MacCormick (1974). As a result of this double demand usage, forecasts by retrospective simulation are biased towards lower total cost.

The purpose of this report is to investigate the performance of other forecasting techniques designed to decrease prediction bias and/or dispersion. The forecasting performance of the retrospective simulation method will be used as a bench-mark to gauge improvements obtained by other methods.

## 1.5 Experimental Design

In our statistical policy simulations we have chosen to examine a 16-item system with the parameter settings of Table 1.1 . The system is

a subset of the 72-item negative binomial system used in previous studies [Ehrhardt (1976), MacCormick (1974)]. We have chosen to examine only those items with the highest variance-to-mean ratio and shortest revision interval since these parameter settings produced the largest forecast biases in those studies.

TABLE 1.1

SYSTEM PARAMETERS

| Factor | Levels | Number of Levels in Each System |
|---|---|---|
| Demand Distribution | Negative Binomial ($\sigma^2/\mu = 9$) | 1 |
| Mean Demand, $\mu$ | 2, 8 | 2 |
| Unit Holding Cost, h | 1 | 1 |
| Unit Backlog Penalty Cost, p | 4, 99 | 2 |
| Replenishment Setup Cost, k | 32, 64 | 2 |
| Replenishment Leadtime, L | 0, 4 | 2 |
| Revision Interval | 13 | 1 |

In this study we discuss forecasts of system-wide operating characteristics, and of individual item characteristics. System characteristics are defined as

the sum of the individual characteristics for holding quantity, backlog quantity, and total cost, and as the average of the individual characteristics for replenishment frequency and backlog frequency.

The performance of the retrospective simulation forecasting method on our 16-item inventory system is summarized in Table 1.2 . Notice that, on the average, forecasts underestimate each average operating characteristic value. The forecasts of holding quantity and replenishment frequency incur small percentage errors; however, backlog quantity, backlog frequency, and total cost have percentage errors of 46.7%, 23.3%, and 29.9% respectively. Most of the bias in forecasting total cost is due to the extreme under-estimation of backlog quantity.

Single-item data appear in Appendix A. Notice that in forecasting backlog quantity and backlog frequency the largest forecasting errors occur for those items with a high penalty cost. This is due to the fact that a high penalty cost will cause the policy parameters to be set in such a way that backlogs are rare events. Forecasting the occurrence of these rare events is difficult, especially when using retrospective simulation which uses the same demands to set the policy parameters and again to forecast the system performance. In fact for several items with high penalty costs no backlogs were ever predicted. In forecasting total cost note that, in addition to high penalty cost items, those items with small means have higher forecasting biases. The level of dispersion in the bias for forecasting total cost is higher for those items with high penalty costs and large means.

TABLE 1.2

Retrospective Simulation Forecasting Performance on a 16-Item System

| OPERATING CHARACTERISTIC | AVERAGE ACTUAL VALUE | AVERAGE FORECASTED VALUE | BIAS (LEVEL OF DISPERSION) | PERCENTAGE UNDERESTIMATES |
|---|---|---|---|---|
| HOLDING QUANTITY | 416.8 | 410.0 | 6.8 ( 27.4) | 1.6 |
| BACKLOG QUANTITY | 19.1 | 10.2 | 8.9 ( 6.3) | 46.7 |
| BACKLOG FREQUENCY | .1126 | .0864 | .0262 ( .029) | 23.3 |
| REPLENISHMENT FREQUENCY | .1519 | .1470 | .0049 ( .020) | 3.3 |
| TOTAL COST | 806.3 | 565.1 | 241.3 (117.5) | 29.9 |

## 2. FORECASTING BY APPROXIMATE EXPRESSIONS FOR OPERATING CHARACTERISTICS

Ehrhardt (1977) developed a number of expressions that give approximate values for the following operating characteristics of (s,S) inventory systems:

$H \equiv$ average holding cost per period

$B \equiv$ average backlog cost per period

(10) $P \equiv$ backlog protection, i.e., frequency of periods without backlogs

$R \equiv$ average replenishment cost per period

$T \equiv$ average total cost per period .

The approximations were derived by using least-squares regression to fit the parameters of functions to the operating characteristics of a large number of parameter settings. These approximations are functions of only the economic parameters, the policy parameters, and the mean and variance of demand. They are accurate to within a few percentage points of the actual values.

In this section we forecast average operating characteristic values by using these formulas with the sample mean $\bar{\xi}$ , and the sample variance $\bar{v}$ of the previous demand history substituted for the unknown mean $\mu$ and variance $\sigma^2$ of the demand distribution.

### 2.1 Forecasts Using Analytical Approximations of Operating Characteristics

Approximations to the exact, analytical expressions for the operating characteristics of fixed, infinite-horizon (s,S) policies were developed by Ehrhardt (1977). He approximated these exact expressions with simplified functions and then fit their parameter values to the observed characteristics of 576 items using least-squares regression. In using these approximations

for forecasting purposes we acknowledge that the approximations were derived for infinite-horizon fixed policies and assume a known mean and variance of demand. Neither situation exists in our model; however, it was hoped that the approximations were robust enough to produce accurate forecasts.

We employ the following approximation formulas from Ehrhardt (1977) to forecast average values of the Power Approximation operating characteristics in (10):

$$H \doteq h(W - .1512\mu + .1684\sigma^2/\mu + .0689)$$

$$P \doteq rG(S|\alpha, \beta) + [(1 - r)/D]\{SG(S|\alpha, \beta) - sG(s|\alpha, \beta) - \alpha\beta G(S|\alpha + 1, \beta) + \alpha\beta G(s|\alpha + 1, \beta)\}$$

$$R \doteq k\mu/(1.003D + .4942\mu + .4990\sigma^2/\mu - .5339)$$

$$T \doteq h(1.110W + .3274D + .4476\sigma^2/\mu - .2234) + p(-.001049W + .003062\sigma^2/\mu) + .3364rk$$

where

$$D = S - s$$

$$r = \mu/(1.003D + .4942\mu + .4990\sigma^2/\mu - .5339)$$

$$W = r[SG(S|\alpha, \beta) - \alpha\beta G(S|\alpha + 1, \beta)] + [(1 - r)/2D]\{x^2G(x|\alpha, \beta) - 2\alpha\beta xG(x|\alpha + 1, \beta) + (\alpha + 1)\alpha\beta^2G(x|\alpha + 2, \beta)\}\Big|_s^S$$

$$G(x|\alpha, \beta) = \begin{cases} [\Gamma(\alpha)\beta^{\alpha}]^{-1} \int_0^x y^{\alpha-1} \exp(-y|\beta)dy \ , & x > 0 \\ 0 & x \le 0 \end{cases}$$

$$\alpha = (L + 1)\mu^2/\sigma^2$$

$$\beta = \sigma^2/\mu \ .$$

Due to the complexity of evaluating the function $G(x|\alpha, \beta)$ in the expressions for $W$ and $P$, we used the alternatives

$$\begin{aligned} W' = r[SN(S_0) - \alpha\beta N(S_1)] + [(1 - r)/2D]\,[S^2N(S_0) \\ - 2\alpha\beta SN(S_1) + (\alpha + 1)\alpha\beta^2N(S_2) - s^2N(s_0) \\ + 2\alpha\beta sN(s_1) - (\alpha + 1)\alpha\beta^2N(s_2)] \end{aligned}$$

and

$$P' = rN(S_0) + [(1 - r)/D]\{SN(S_0) - sN(s_0) - \alpha\beta N(S_1) + \alpha\beta N(s_1)\}$$

where $N(\cdot)$ is the unit Normal Distribution function and

$$S_i = [S - (\alpha + i)\beta]/[(\alpha + i)\beta^2]^{1/2}$$

$$s_i = [s - (\alpha + i)\beta]/[(\alpha + i)\beta^2]^{1/2} \ .$$

Because Ehrhardt's approximations for average backlog cost per period $B$ are inaccurate, we used the relation $B = T - H - R$ for our forecasts. We convert the above operating characteristics to ours by employing the following identities:

| | |
|---|---|
| average holding quantity per period | $\equiv H$ |
| average backlog quantity per period | $\equiv B/p$ |
| backlog frequency | $\equiv 1 - P$ |
| replenishment frequency | $\equiv R/k$ . |

In Table 2.1 we summarize the accuracy of this forecasting technique. All of the operating characteristics were underestimated with the exception of replenishment frequency. In comparison with the retrospective simulation results, Table 1.2, we observe that the prediction bias has been reduced substantially in backlog quantity, from 46.7% to 29.1%, and in backlog frequency, from 23.3% to 8.4%. A slight reduction in bias is also evident in forecasting total cost. However, bias in forecasting holding quantity has increased from 1.6% to 12%, and its level of dispersion has also increased from 27 to 31. The bias dispersion levels for the remaining operating characteristics are essentially the same as those in the retrospective simulation forecasts.

Single item data of the forecasts made using the analytical approximations are given in Appendix B. As mentioned above, bias in forecasting holding quantity increased. The increased bias is particularly evident for those items with low means and high lead times. As before, items with high penalty

TABLE 2.1

Analytic Approximation Formulae Forecasting Performance on a 16-Item System

| OPERATING CHARACTERISTIC | AVERAGE ACTUAL VALUE | AVERAGE FORECASTED VALUE | BIAS (LEVEL OF DISPERSION) | PERCENTAGE UNDERESTIMATES |
|---|---|---|---|---|
| HOLDING QUANTITY | 421.5 | 371.1 | 50.5<br>( 31.1) | 12.0 |
| BACKLOG QUANTITY | 18.9 | 13.4 | 5.5<br>( 6.4) | 29.1 |
| BACKLOG FREQUENCY | .1120 | .1025 | .0095<br>( .022) | 8.4 |
| REPLENISHMENT FREQUENCY | .1513 | .1575 | -.0062<br>( .019) | -4.1 |
| TOTAL COST | 800.9 | 598.4 | 202.5<br>(117.9) | 25.3 |

costs experienced the largest percentage errors in forecasting backlog quantity and total cost. For these same operating characteristics, prediction bias is even greater for those items with low mean values.

### 2.2 Forecasts Using Multiplicative Approximations of Operating Characteristics

Approximate expressions of system operating characteristics based on multiplicative functional forms were derived by Ehrhardt (1977). The parameters of the functions were fit to the observed characteristics of a large multi-item inventory system. These approximations are much easier to evaluate and interpret than the analytical approximations discussed in Section 2.1. We considered two sets of approximations; one set derived for fixed policies and the other for statistical policies.

#### 2.2.1 Multiplicative Approximations for Fixed Policies

In this section forecasts were made by multiplicative approximations for fixed power approximation policy operating characteristics. These approximations suffer from the same limitations as the analytic approximations, in that they assume an infinite-horizon constant policy and a known mean and variance of the demand distribution. We use the following expressions from Ehrhardt (1977) for the operating characteristics in (10)

$$\begin{aligned} H \doteq 2.598h \exp(.1204\sigma^2/\mu)\mu^{.5076 - .004964\sigma^2/\mu} \\ (L+1)^{.2287 + .01472\sigma^2/\mu - .0900\mu/\sigma^2} \\ (p/h)^{-3.040h/p + .01137\sigma^2/\mu - .07292\mu/\sigma^2 + .03355\left(\frac{\sigma^2 h}{\mu p}\right)} \\ (k/h)^{.3057 - .02667\sigma^2/\mu + .03058\mu/\sigma^2} \end{aligned}$$

$$P \doteq 49.74(p/h)^{-7.353h/p}\exp(.01041p/h)/(1 + p/h)$$

$$R \doteq .5763h(\sigma^2/\mu)^{-.2774}\mu^{.4959 + .007906\sigma^2/\mu - .003856\mu/\sigma^2}$$

$$(L + 1)^{-.05744}(k/h)^{.6046 - .06563\mu/\sigma^2}$$

$$T \doteq 2.438h(\sigma^2/\mu)^{.3836}\mu^{.4964 - .005209\sigma^2/\mu}$$

$$(p/h)^{-.9230h/p + .01508\sigma^2/\mu}$$

$$(L + 1)^{.1498 + .01231\sigma^2/\mu - .07050\mu/\sigma^2}$$

$$(k/h)^{.3095 - .01310\sigma^2/\mu + .1073\mu/\sigma^2}$$

$$B = T - H - R \; .$$

Table 2.2 summarizes the forecasting performance of these approximations. As with the analytical approximation method, all the operating characteristics were underestimated with the exception of replenishment frequency. In comparison with the retrospective simulation method, Table 1.2, we discover a reduction in the percentage forecasting error for backlog quantity, backlog frequency, and total cost of 6.2%, 11.6%, and 4.4% respectively. The reductions are smaller than those obtained by the analytical approximations, Table 2.1. The difference is most noticeable in forecasting backlog quantities for which the analytical approximations achieved a 17.6% reduction. Although the multiplicative approximation procedure's bias in forecasting holding quantity was less than that of the analytical approximations, 7% as compared to 12%, the level of dispersion in the bias was almost double. Levels of dispersion for the other operating characteristics are similar to those

TABLE 2.2

Fixed Policy Multiplicative Formulae Forecasting Performance on a 16-Item System

| OPERATING CHARACTERISTIC | AVERAGE ACTUAL VALUE | AVERAGE FORECASTED VALUE | BIAS (LEVEL OF DISPERSION) | PERCENTAGE UNDERESTIMATES |
|---|---|---|---|---|
| HOLDING QUANTITY | 421.5 | 392.4 | 29.1 ( 55.6) | 6.9 |
| BACKLOG QUANTITY | 18.9 | 11.3 | 7.7 ( 5.7) | 40.5 |
| BACKLOG FREQUENCY | .1120 | .0989 | .0131 ( .024) | 11.7 |
| REPLENISHMENT FREQUENCY | .1513 | .1591 | -.0078 ( .019) | -5.2 |
| TOTAL COST | 800.9 | 596.7 | 204.2 (122.7) | 25.5 |

obtained by the previous methods.

Appendix C gives the individual item data of forecasts made by the fixed-policy multiplicative approximation method. Notice that the forecasts of backlog quantity for every item were below the actual realizations and, as before, those items with large penalty costs had the highest percentage differences. For each operating characteristic the forecasts for those items with low means, long leadtimes, and high penalty costs had the largest percentage forecasting errors.

2.2.2 Multiplicative Approximations for Statistical Policies

Ehrhardt also derived multiplicative approximations for the operating characteristics of the Statistical Power Approximation. The approximations were derived only for multi-item systems having fixed variance-to-mean ratios, so using these expressions is tantamount to assuming that the variance-to-mean ratio is known with certainty. This approach avoids the shortcomings of the others in that the approximations were derived for statistical policies but they still require the substitution of our statistical estimate of mean demand in place of the actual mean.

We used the following approximations for the characteristics in (10). The expressions require a variance-to-mean ratio of 9 and a 13-period revision interval.

$$H \doteq 10.76h\mu^{.4499}(L + 1)^{.4187}(p/h)^{-3.691h/p}(k/h)^{.1028}$$

$$R \doteq .2676h\mu^{.5688}(L + 1)^{-.1084}(k/h)^{.6470}$$

$$P \doteq (-.0057 + .9853p/h) / (1 + p/h)$$

$$T \doteq 3.776h\mu^{.4387}(L + 1)^{.3346}(p/h)^{.2727}(k/h)^{.1843}$$

$$B = T - H - R .$$

A summary of the results of forecasting system performance using these approximations appears in Table 2.3. The results clearly demonstrate how accurate these approximations are when only an estimate of the mean demand is required. Backlog quantity, backlog frequency, and total cost are forecasted with prediction errors of 4.8, 6.0, and 3.8 percent respectively, compared with 46.6, 23.3, and 29.9 percent prediction errors of the same operating characteristics when forecasting by retrospective simulation. With the exception of holding quantity, the levels of bias dispersion for the remaining operating characteristics were slightly less than those obtained by retrospective simulation. Note also that both backlog quantity and backlog frequency were overestimated. This contrasts with the severe underestimation present when retrospective simulation or the previous approximation formulas are used.

Single-item data appear in Appendix D. The most severe biases in forecasting total cost are for those items with low means and high penalty costs. The highest levels of bias dispersion are for those items with large penalty cost parameters.

Although the forecasts in this section are accurate, it must be remembered that we have assumed a knowledge of the variance-to-mean ratio,

TABLE 2.3

Statistical Policy Multiplicative Approximation Formulae Forecasting Performance on a 16-Item System

| OPERATING CHARACTERISTIC | AVERAGE ACTUAL VALUE | AVERAGE FORECASTED VALUE | BIAS (LEVEL OF DISPERSION) | PERCENTAGE UNDERESTIMATES |
|---|---|---|---|---|
| HOLDING QUANTITY | 421.5 | 403.4 | 18.1 ( 45.2) | 4.3 |
| BACKLOG QUANTITY | 18.9 | 19.8 | -0.9 ( 5.9) | -4.8 |
| BACKLOG FREQUENCY | .1120 | .1187 | -.0067 ( .024) | -6.0 |
| REPLENISHMENT FREQUENCY | .1513 | .1502 | .0011 ( .019) | .7 |
| TOTAL COST | 800.9 | 770 | 30.3 (108.9) | 3.8 |

a situation one will not encounter in practice. It is clear that great forecasting improvements can be made if estimates of demand variance are improved.

The results suggest that it may be promising to investigate operating characteristic formulas for statistical policies that include the variance-to-mean ratio as a variable if and when these formulas are derived.

## 3. FORECASTING BY A FUTURE DEMAND GENERATION TECHNIQUE

The bias in the forecasts produced by the retrospective simulation procedure can be attributed mainly to the double use of the demand information; once to fix the policy parameters (s,S) and then again to predict the system's performance. In this section we examine a forecasting procedure which differs from the retrospective simulation method in that, instead of subjecting the policy parameters to the previous demand history, we subject them to a sequence of forecasted demands.

The sequence of forecasted demands consists of $n$ independent demands randomly generated from a postulated demand distribution whose parameters are estimated from the sample statistics of the previous demand history. An average value for each of the system's operating characteristics is then obtained by exposing the newly revised policy to the forecasted demand sequence. This entire procedure is repeated until we have accumulated $m$ simulated values for each operating characteristic. The forecasted value of each operating characteristic is taken as the average of the $m$ predictions. We shall refer to $m$ as the number of replications. Note that although we have eliminated the explicit sequence of previous demands in our forecasting simulations, the method still requires a double use of demand information: once to set the policy parameters, and then to estimate the simulated demand parameters.

Recall that in our experiments the underlying demand distribution is negative binomial with a variance-to-mean ratio of 9. For this reason, we examined the future demand generation forecasting technique with postulated demand distributions

1) negative binomial (Section 3.1)

and

2) gamma (Section 3.2) .

The gamma was chosen because it is the continuous analogue of the negative binomial distribution. By postulating a specific form of the demand distribution, serious forecasting errors may result if, in fact, the underlying demand distribution is grossly different. In our investigation of the future demand generation technique, we did not address this issue. Nonetheless, it may be worthwhile to investigate the technique's robustness.

### 3.1 Forecasts by Future Demand Generation with a Postulated Negative Binomial Demand Distribution

Let $\phi(\xi)$, $\xi = 0, 1, 2, \ldots$, be the probability distribution of demand per period. The mathematical form of the negative binomial distribution is

$$\phi(\xi) = \binom{r - 1 + \xi}{r - 1} q^r p^\xi , \qquad p + q = 1, \quad p, q \geq 0$$

with expected value $\mu = rp/q$ , variance $\sigma^2 = rp/q^2$ and variance-to-mean ratio $\sigma^2/\mu = 1/q$ .

We utilize the method of moments to estimate the parameters $p$ and $r$. If $t$ is the first period in a revision interval and $\xi_{t-1}, \xi_{t-2}, \ldots, \xi_{t-T}$ is the previous demand history, we calculate the solutions $\hat{p}, \hat{r}$ of the equations

$$(11) \qquad \overline{\xi} = \hat{r}\,\hat{p}/(1 - \hat{p}) \quad \text{and} \quad \overline{v} = \hat{r}\,\hat{p}/(1 - \hat{p})^2$$

where $\bar{\xi}$ and $\bar{v}$ are the sample mean and variance, defined in (7) . We solve for $\hat{p}$ and $\hat{r}$ to obtain

$$(12) \quad \hat{p} = 1 - \bar{\xi}/\bar{v} \quad \text{and} \quad \hat{r} = \bar{\xi}^2/(\bar{v} - \bar{\xi}) \ .$$

Forecasts are made by subjecting the revised policy to a sequence of $n$ demands generated from a negative binomial distribution with parameters $\hat{p}$ and $\hat{r}$ . To be consistent with previous results, the value of $T$ chosen was 13, i.e., only the 13 most recent demands were employed in computing $\hat{p}$ and $\hat{r}$ .

Before an assessment of the forecasting accuracy of this method can be made, the size of the generated demand sequence $n$, and the number of replications $m$ must be set.

Table 3.1 was constructed to demonstrate the sensitivity of the bias to the value of $n$ . As $n$ increases we note a decrease in the bias of forecasts of backlog quantity, backlog frequency, and replenishment frequency, together with a reduction in the bias dispersion for each operating characteristic. In particular, the dispersion of the bias for total cost was reduced by roughly 30% when $n$ was increased from 13 to 100. However, increasing $n$ also results in an enormous increase in the bias of holding quantity forecasts, with a corresponding rise in the bias for total cost. This increase is due to less significant transient effects when the policy is operated over a prolonged interval. To illustrate, let $y$ be the inventory on hand at the end of any period and suppose that upon revision of $(s,S)$ to values, say $(s',S')$, $y$ is greater than $S'$. Let $k$ be the number of periods that $y$ is above $S'$, and note that

TABLE 3.1

Sensitivity of Bias to the Length of the Demand Sequence
Forecasts made by Future Demand Generation Technique
with a Single Replication

| OPERATING CHARACTERISTIC | LENGTH OF DEMAND SEQUENCE | AVERAGE ACTUAL VALUE | AVERAGE FORECASTED VALUE | BIAS (LEVEL OF DISPERSION) | PERCENTAGE UNDERESTIMATES |
|---|---|---|---|---|---|
| HOLDING QUANTITY | 13 | 416.8 | 416.7 | 0.1 ( 36.7) | 0.023 |
| | 100 | | 379.7 | 37.1 ( 28.6) | 8.9 |
| BACKLOG QUANTITY | 13 | 19.1 | 12.5 | 6.6 ( 8.5) | 34.6 |
| | 100 | | 13.3 | 5.8 ( 6.7) | 30.4 |
| BACKLOG FREQUENCY | 13 | .1126 | .0895 | .0231 ( .033) | 20.5 |
| | 100 | | .0964 | .0162 ( .025) | 14.4 |
| REPLENISHMENT FREQUENCY | 13 | .1519 | .1474 | .0045 ( .023) | 3.0 |
| | 100 | | .1553 | -.0034 ( .019) | -2.2 |
| TOTAL COST | 13 | 806.3 | 622.7 | 183.6 (170.2) | 22.8 |
| | 100 | | 593.1 | 213.2 (122.4) | 26.4 |

once $y$ falls below $S'$ it will not exceed $S'$ in any of the following periods. As the ratio of $k$ to the number of periods that $(s',S')$ is in effect decreases, the effect of those $k$ periods of high inventory levels on the average inventory level is decreased. Hence, increasing $n$ leads to underestimation of actual holding quantities. Because of this degradation, $n$ was set equal to the revision interval length (13 periods).

The effect of increasing the number of replications on prediction bias is shown in Table 3.2. Observe that increasing $m$ from one to eight results in a substantial decrease in bias dispersion with only a minor change in the percentage forecasting bias for each operating characteristic. Further increasing $m$ shows only a slight decrease in bias dispersion and, in fact, a small increase in prediction bias.

We summarize in Table 3.3 the system performance of the future demand generation forecasting method with $n$ and $m$ equal to 13 and 8, respectively. Notice that the forecast of holding quantity is extremely accurate, however, as with preceding methods, forecasts of backlog frequency, backlog quantity (and thus total cost), have large biases. In comparison with the retrospective simulation technique we observe a smaller prediction bias for every operating characteristic. Bias dispersion levels are approximately the same for both procedures.

Appendix E contains the single-item forecasting data. Note that on the average forecasts of backlog quantity were below the average actual realizations for every item. As before, percentage forecasting errors of backlog quantity, backlog frequency, and total cost, tend to increase with increases in the penalty cost and the leadtime, and with decreases in the mean.

Much of the bias in the forecasts produced by the future demand generation forecasting procedure is the outgrowth of a double use of the

TABLE 3.2

Sensitivity of Bias to the Number of Replications. Forecasts made using Future Demand Generation Technique with demand sequence of length $n = 13$.

| OPERATING CHARACTERISTIC | AVERAGE ACTUAL VALUE | NUMBER OF REPLICATIONS | AVERAGE FORECASTED VALUE | BIAS (LEVEL OF DISPERSION) | PERCENTAGE UNDERESTIMATES |
|---|---|---|---|---|---|
| HOLDING QUANTITY | 416.8 | 1 | 416.7 | 0.1 (36.6) | .023 |
| | | 8 | 417.1 | -0.3 (29.7) | -0.65 |
| | | 20 | 417.9 | -1.0 (29.0) | - .245 |
| BACKLOG QUANTITY | 19.1 | 1 | 12.5 | 6.6 ( 8.5) | 34.6 |
| | | 8 | 12.7 | 6.4 ( 6.6) | 33.7 |
| | | 20 | 12.7 | 6.5 ( 6.5) | 33.8 |
| BACKLOG FREQUENCY | .1126 | 1 | .0895 | .0231(.033) | 20.5 |
| | | 8 | .0894 | .0232(.026) | 20.6 |
| | | 20 | .0882 | .0244(.025) | 21.7 |
| REPLENISHMENT FREQUENCY | .1519 | 1 | .1474 | .0045(.023) | 3.0 |
| | | 8 | .1473 | .0046(.019) | 3.1 |
| | | 20 | .1472 | .0047(.019) | 3.1 |
| TOTAL COST | 806.3 | 1 | 622.7 | 183.6 (170.2) | 22.8 |
| | | 8 | 617.1 | 189.2 (121.2) | 23.5 |
| | | 20 | 613.9 | 192.4 (119.2) | 23.9 |

TABLE 3.3

Future Demand Generation Forecasting Performance on a 16-Item System with $n = 13$ and $m = 8$.

| OPERATING CHARACTERISTIC | AVERAGE ACTUAL VALUE | AVERAGE FORECASTED VALUE | BIAS (LEVEL OF DISPERSION) | PERCENTAGE UNDERESTIMATES |
|---|---|---|---|---|
| HOLDING QUANTITY | 416.8 | 417.1 | -0.3<br>( 29.7) | -0.65 |
| BACKLOG QUANTITY | 19.1 | 12.7 | 6.4<br>( 6.6) | 33.7 |
| BACKLOG FREQUENCY | .1126 | .0894 | .0232<br>( .026) | 20.6 |
| REPLENISHMENT FREQUENCY | .1519 | .1473 | .0046<br>( .019) | 3.1 |
| TOTAL COST | 806.3 | 617.1 | 189.2<br>(121.2) | 23.5 |

demand statistics; the same statistics being used to fix the policy parameters (s,S) and to estimate demand distribution parameters. This demand usage is less direct than that employed by retrospective simulation, and as a result forecast bias is reduced.

### 3.1.1 Future Demand Generation Forecasting Technique Using Variable Means and Variances

In Section 3.1 it was pointed out that the future demand generation technique produced biased forecasts, although less biased than those obtained with retrospective simulation. In this section we discuss a sampling procedure designed to further reduce the undesirable side effects caused by double use of demand information.

We explicitly acknowledge the uncertainty about the demand distribution parameters by estimating their probability distributions and then sampling from these distributions to obtain estimates of the parameters of the demand distribution. The estimated parameters are then used to generate a sequence of forecasted demands.

We construct an approximate probability distribution for the demand mean $\mu$ by noting that the sample mean $\bar{x}$ of a sequence of $n$ independent, identically distributed random variables has an asymptotic normal distribution. Hence, we sample for $\mu$ from a normal distribution with mean $\bar{\xi}$ and variance $\bar{v}/n$, where $\bar{\xi}$ and $\bar{v}$ are as defined in (7). Asymptotic theory also tells us that if we assume $\bar{\xi}$ and $\bar{v}$ are independent, then the distribution of $\bar{v}$ will approach a chi-square distribution as $n$ increases. Thus, we sample for the demand variance $\sigma^2$ from a chi-square distribution with $n - 1$ degrees of freedom.

For each replication s of the future demand generation technique we use the above sampling procedure to obtain estimates $\hat{\mu}_s$ and $\hat{\sigma}^2_s$ of the mean and variance of the demand distribution. Our forecasted demand sequence is then generated from a negative binomial distribution with parameters $\hat{p}_s = 1 - \hat{\mu}_s/\hat{\sigma}^2_s$ and $\hat{r}_s = \hat{\mu}^2_s/(\bar{v}_s - \hat{\mu}_s)$. Therefore, each replication in our forecasting procedure uses demands generated from a probability distribution which is itself generated from a postulated distribution.

Table 3.4 summarizes the forecasting performance of this method. In comparison with the same procedure without sampling for demand parameters, Table 3.3, we note a reduction in forecasting bias for all operating characteristics except holding quantity which has increased slightly. Of particular importance is the fact that the percentage underestimation of backlog quantity has been reduced from 33.7% to 22.3%, and that of backlog frequency from 20.6% to 15.0%. These improvements are accompanied with only a small increase in bias dispersion.

Single-item data appear in Appendix F. Sensitivity to parameter settings is analogous to that observed without sampling for demand means and variances.

## 3.2 <u>Forecasts by Future Demand Generation with a Postulated Gamma Demand Distribution</u>

In this section we examine the forecasting performance of the future demand generation technique when the demands are generated from a Gamma distribution whose parameters are estimated using sample statistics of the previous demand history. Since the gamma is a continuous distribution, the

TABLE 3.4

Performance of the Future Demand Generation Technique
Using Variable Means and Variances

| OPERATING CHARACTERISTIC | AVERAGE ACTUAL VALUE | AVERAGE FORECASTED VALUE | BIAS (LEVEL OF DISPERSION) | PERCENTAGE UNDERESTIMATES |
|---|---|---|---|---|
| HOLDING QUANTITY | 416.8 | 423.4 | -6.5 ( 31.1) | -1.6 |
| BACKLOG QUANTITY | 19.1 | 14.8 | 4.3 ( 7.3) | 22.3 |
| BACKLOG FREQUENCY | .1126 | .0957 | .0169 ( .026) | 15.0 |
| REPLENISHMENT FREQUENCY | .1519 | .1486 | .0033 ( .020) | 2.2 |
| TOTAL COST | 806.3 | 639.5 | 166.8 (124.4) | 20.7 |

demands generated from it constitute a sequence of nonnegative real numbers. The generated demands were not rounded-off to the nearest integer.

The gamma is a two-parameter distribution, denoted by $G(\alpha,\beta)$, where $\alpha$ and $\beta$ are positive and real. If $X$ is a random variable having a gamma distribution, then the probability density function of $X$ is

$$f_X(x) = \begin{cases} x^{\alpha-1}\, e^{-x/\beta} \,/\, \beta^{\alpha}\, \Gamma(\alpha) & 0 \le x \le \infty \\ 0 & \text{elsewhere} \end{cases}$$

with expectation $\mu = \alpha\,\beta$, variance $\sigma^2 = \alpha\,\beta^2$, and variance-to-mean ratio $\sigma^2/\mu = \beta$.

As was the case for the negative binomial distribution, we use the method of moments to obtain estimates of the parameters $\alpha$ and $\beta$. Thus, if $t$ is the first period of a revision interval and $\xi_{t-1}, \xi_{t-2}, \ldots, \xi_{t-T}$ is the previous demand history, we calculate the solutions $\hat{\alpha}, \hat{\beta}$ of the equations

$$(13) \qquad \overline{\xi} = \hat{\alpha}\,\hat{\beta} \qquad \text{and} \qquad \overline{v} = \hat{\alpha}\,\hat{\beta}^2 \;,$$

where $\overline{\xi}$ and $\overline{v}$ are as defined in (7).

Hence

$$(14) \qquad \hat{\alpha} = \overline{\xi}^2/\overline{v} \qquad \text{and} \qquad \hat{\beta} = \overline{v}/\overline{\xi} \;.$$

Forecasts are made at the beginning of each revision interval by subjecting the revised control policies to a sequence of $n$ independent

demands generated from a $G(\hat{\alpha}, \hat{\beta})$ distribution. In keeping with the practice employed in the other forecasting methods, only the 13 most recent demands were used in computing $\bar{\xi}$ and $\bar{v}$ .

A system performance summary of this forecasting method for single and multiple replications appears in Table 3.5. Notice that forecasts of backlog quantity and backlog frequency have negative biases which contrasts with the positive biases produced by the forecasting methods discussed previously. Again we see that increasing the number of replications decreases the dispersion of the bias. In particular, bias dispersion for backlog quantity and total cost has been reduced approximately 30% by increasing m from 1 to 10. A comparison of this procedure, using ten replications, with the retrospective simulation method of forecasting gives several interesting observations. First, the percentage forecasting bias for backlog quantity is 5.2% for this method as compared with the gross 46.6% bias by retrospective simulation forecasting. Second, the improved forecasts of backlog quantity lead to a smaller bias in forecasting total cost, 11% as compared with almost 30% by retrospective simulation. Third, the dispersions are greater, most notably for the total cost forecast which has a dispersion of 185 as compared to 117 for retrospective simulation.

Appendix G gives the individual-item forecasts using ten replications. Forecasting bias for total cost tends to increase as leadtime and penalty cost increase, and decrease as the mean increases. Note also that the dispersion of the bias for total cost is highly sensitive to penalty cost, and is generally greater for those items with a high setup cost.

TABLE 3.5

Future Demand Generation Forecasting Performance with Demands Generated from an Estimated Gamma Distribution n=13

| OPERATING CHARACTERISTIC | AVERAGE ACTUAL VALUE | NUMBER OF REPLICATIONS | AVERAGE FORECASTED VALUE | BIAS (DISPERSION) | PERCENTAGE UNDERESTIMATES |
|---|---|---|---|---|---|
| HOLDING QUANTITY | 416.8 | 1 | 400.0 | 16.9 ( 40.8) | 4.0 |
| | | 10 | 399.4 | 17.4 ( 35.0) | 4.2 |
| BACKLOG QUANTITY | 19.1 | 1 | 19.7 | -.6 ( 11.0) | -3.2 |
| | | 10 | 20.1 | -1.0 ( 8.5) | -5.3 |
| BACKLOG FREQUENCY | .1126 | 1 | .1266 | -.0139(.039) | -12.4 |
| | | 10 | .1274 | -.0148(.031) | -13.1 |
| REPLENISHMENT FREQUENCY | .1519 | 1 | .1558 | -.0038(.024) | -2.5 |
| | | 10 | .1562 | -.0043(.022) | -2.8 |
| TOTAL COST | 806.3 | 1 | 712.0 | 94.3 (238.3) | 11.7 |
| | | 10 | 716.6 | 89.7 (184.7) | 11.1 |

The changes in forecasting performance resulting from the use of gamma distribution in place of the negative binomial are greater than we had expected. We can only speculate that the continuous property of the gamma distribution is responsible.

## 4. COMPARISON OF FORECASTING PROCEDURES

In this section we compare the performance of the forecasting procedures discussed in the preceding sections. We also suggest topics for future research.

### 4.1 Properties of Single-item Forecasts

As a measure of the deviation between forecasted values $f$, and actual realizations $a$, we define the mean square error of a single-item forecast as

$$MSE \equiv E(a - f)^2 ,$$

and the root mean square error as

$$RMSE \equiv \sqrt{MSE} .$$

Using the fact that

$$Var(a - f) = E(a - f)^2 - [E(a - f)]^2 ,$$

and the definitions of bias $B$ and dispersion $D$ given in (8) and (9), we have

$$(15) \qquad MSE = B^2 + D^2 .$$

Hence, a forecasting scheme's MSE becomes larger with increases in bias and/or dispersion.

Previous studies [Ehrhardt (1976), MacCormick (1974)] have indicated that for single-item forecasts using retrospective simulation, the dispersion

term dominates the MSE. In fact, the dispersion is so large that the forecasting method is essentially useless for single items. In our study we have observed this phenomenon. For not only the retrospective simulation method, but for the other methods investigated as well. This fact is exhibited by the single-item data in Appendices A through G.

In the next section we examine the impact of dispersion on the MSE when the single item forecasts of each operating characteristic are aggregated in a multi-item system.

## 4.2 Properties of Multi-item Forecasts

In the previous section we observed that for single-item forecasts the dispersion overwhelms the bias, and this was true for each of the forecasting methods discussed in this study. However, we will show that the dispersion becomes small relative to the bias when forecasts are aggregated in a multi-item system containing a sufficiently large number of items with independent demand processes.

To illustrate, consider a system consisting of $n$ independent items. Let $b_i$ and $d_i$ represent, respectively, the bias and dispersion for item $i$. Thus, when single-item forecasts are aggregated over the multi-item system, the aggregate bias $B$ and dispersion $D$ are given by

$$B = \sum_{i=1}^{n} b_i \qquad \text{and} \qquad D = \left( \sum_{i=1}^{n} d_i^2 \right)^{1/2} .$$

The mean square error of the multi-item forecast can be expressed as

$$\text{(16)} \qquad \text{MSE} \equiv B^2 + D^2 = \left( \sum_{i=1}^{n} b_i \right)^2 + \sum_{i=1}^{n} d_i^2 .$$

If we let

$$(17) \qquad \overline{b} = n^{-1} \sum_{i=1}^{n} b_i \qquad \text{and} \qquad \overline{d} = n^{-1} \sum_{i=1}^{n} d^2_i \ ,$$

then (16) becomes

$$\text{MSE} \equiv n^2\overline{b}^2 + n\overline{d} \ .$$

We assume that $b_i$ and $d_i$ are finite for all $i$. Therefore, we observe that for sufficiently large values of $n$ and $\overline{b} \neq 0$, the dominant factor in the MSE of a multi-item forecast is the bias.

When comparing forecasting procedures for particular operating characteristic the above discussion suggests that the procedure which exhibits the smallest MSE in our 16-item system will not necessarily have the smallest MSE for any system size.

To illustrate, consider a hypothetical system consisting of $m$ independent 16-item systems, each identical to ours. Suppose that in our 16-item system forecasting procedure $A$ has bias and dispersion quantities $\overline{b}_A$ and $\overline{d}_A$ as defined in (17). Let forecasting procedure $B$ be characterized by bias and dispersion quantities $\overline{b}_B$ and $\overline{d}_B$, defined in the same way. Further suppose that $\overline{b}_A > \overline{b}_B$ and $\overline{d}_A < \overline{d}_B$. Then for any $m$ we have

$$\text{MSE}_A - \text{MSE}_B = m^2(\overline{b}^2{}_A - \overline{b}^2{}_B) + m(\overline{d}^2{}_A - \overline{d}^2{}_B) \ .$$

Let $\alpha = (\overline{b}^2_A - \overline{b}^2_B)$ and $\beta = (\overline{d}^2_A - \overline{d}^2_B)$ . Then

$$MSE_A - MSE_B = \alpha m^2 + \beta m$$

is quadratic in $m$ with roots at 0 and at $-\beta/\alpha$ . Thus, for every $m$ less than $m^* = -\beta/\alpha$ method A will have lower MSE than method B, and method B will have lower MSE than method A for every $m$ greater than $m^*$. We can interpret $m^*$ as the forecasting system size for which the better method changes.

Tables 4.1 and 4.2 provide a summary of the performance of the forecasting procedures disucssed in this study. The entries in Table 4.1 are the bias, dispersion and root mean square error of each operating characteristic forecast, and Table 4.2 has these same entries expressed as the percentage of the actual value. Observe that for holding quantity the retrospective simulation method has the smallest RMSE, but the future demand generation technique with postulated negative binomial demand distribution has the smallest bias: Therefore, for sufficiently large systems the latter method will dominate. For backlog frequency the analytic approximation method has the smallest bias and lowest dispersion, and is, therefore, superior for all system sizes. For backlog quantity and replenishment frequency the analytic approximation method also has the lowest RMSE; however, the smallest biases for these operating characteristics are produced by the future demand generation method with postulated negative binomial demand distribution. We, therefore, can conclude that in larger systems this method will dominate the analytic approximation technique. Finally, for total cost the future demand generation technique with postulated gamma demand distribution has the smallest RMSE, but it has a

TABLE 4.1

Comparison of Forecasting Methods' Performance on a 16-item Inventory System

| FORECASTING METHOD | HOLDING QUANTITY | BACKLOG QUANTITY | BACKLOG FREQUENCY | REPLENISHMENT FREQUENCY | TOTAL COST |
|---|---|---|---|---|---|
| RETROSPECTIVE SIMULATION | 6.8 (27.4) [28.2] | 8.9 (6.3) [10.9] | .0262 (.029) [.0391] | .0049 (.020) [.0206] | 241.3 (117.5) [268.4] |
| ANALYTIC APPROXIMATION FORMULAE | 50.5 (31.1) [59.3] | 5.5 (6.4) [ 8.4] | .0095 (.022) [.0239] | -.0062 (.019) [.0200] | 202.5 (117.9) [234.3] |
| FIXED POLICY MULTIPLICATIVE APPROXIMATIONS | 29.1 (55.6) [62.8] | 7.7 (5.7) [ 9.6] | .0131 (.024) [.0274] | -.0078 (.019) [.0205] | 204.2 (122.7) [238.2] |
| FUTURE DEMAND GENERATION (NEGATIVE BINOMIAL) | -6.5 (31.1) [31.8] | 4.3 (7.3) [ 8.5] | .0169 (.026) [.0310] | .0033 (.020) [.0203] | 166.8 (124.4) [208.1] |
| FUTURE DEMAND GENERATION (GAMMA) | 17.4 (35.0) [39.1] | -1.0 (8.5) [ 8.6] | -.0148 (.031) [.0344] | -.0043 (.022) [.0224] | 89.7 (184.7) [205.3] |

Entries are: Bias (Dispersion)
[RMSE]

TABLE 4.2

Comparison of Forecasting Methods' Performance on a 16-item System

| FORECASTING METHOD | HOLDING QUANTITY | BACKLOG QUANTITY | BACKLOG FREQUENCY | REPLENISHMENT FREQUENCY | TOTAL COST |
|---|---|---|---|---|---|
| RETROSPECTIVE SIMULATION | 1.6 ( 6.6) [ 6.8] | 46.6 (33.0) [57.1] | 23.3 (25.8) [34.7] | 3.3 (13.2) [13.6] | 29.9 (14.6) [33.3] |
| ANALYTIC APPROXIMATION FORMULAE | 12.0 ( 7.4) [14.1] | 29.1 (33.9) [44.4] | 8.4 (19.6) [21.3] | -4.1 (12.6) [13.2] | 25.3 (14.7) [29.3] |
| FIXED POLICY MULTIPLICATIVE APPROXIMATIONS | 6.9 (13.2) [14.9] | 40.5 (30.2) [50.8] | 11.7 (21.5) [24.5] | -5.2 (12.6) [13.5] | 25.5 (15.3) [29.7] |
| FUTURE DEMAND GENERATION (NEGATIVE BINOMIAL) | -1.6 ( 7.5) [ 7.6] | 22.3 (38.2) [44.5] | 15.0 (23.1) [27.5] | 2.2 (13.2) [13.5] | 20.7 (15.4) [25.8] |
| FUTURE DEMAND GENERATION (GAMMA) | 4.2 ( 8.4) [ 9.4] | -5.3 (44.5) [45.0] | -13.1 (27.5) [30.6] | -2.8 (14.5) [14.7] | 11.1 (22.9) [25.5] |

Entries are: Bias (Dispersion)
[RMSE]
expressed as percentage of actual

higher level of dispersion than any of the other methods. As a result, we would expect one of the other methods to dominate for systems having fewer items.

In conclusion, we observe that no single method is uniformly superior for each operating characteristics. The two most promising procedures appear to be the analytic approximation method and the future demand generation techniques.

One additional issue that must be considered in selecting a forecasting procedure, is its simplicity of implementation. In this respect the analytic approximations are clearly superior to the future demand generation techniques. The analytic approximation method requires only the computation of a formula for each operating characteristic. Although these formulas are rather complicated, their evaluation requires considerably less effort than the future demand generation techniques which require the generation of many random variates and the explicit simulation of system behavior.

### 4.3 Topics for Future Research

In this section we suggest several topics for future research in forecasting operating characteristic values.

Recall that in Section 2.2.2 we observed that the statistical policy multiplicative approximations perform extremely well, but are not applicable in actual practice since they were derived for a specific variance-to-mean ratio. Consequently, it may be promising to investigate statistical-policy operating characteristic formulas that include the variance-to-mean ratio as a variable if and when these formulas are derived.

Another possible avenue for future research is the investigation of alternative demand estimation methods in the future demand generation

technique. In our research we have used the method of moments. One may be able to obtain better estimates of demand parameters by using other methods. For example, in the case of the negative binomial distribution we might consider the maximum likelihood method.

Further research is also needed in determining the robustness of the future demand generation technique to misspecification of the form of the underlying demand distribution.

A possible improvement in the forecasting performance of the retrospective simulation method may lie in the derivation of an analytical expression for the bias in its forecasts. This expression could then be used as a correction factor for the forecasts. Research along this line is presently underway.

Another possible extension of this study could consider hybrid forecasting procedures, which would use several methods of forecasting simultaneously. In principle, one could then use the best method for each operating characteristic and also choose the best combination of holding cost, backlog cost, and replenishment cost forecasts to obtain a total cost estimate.

Finally we describe a forecasting approach that was mentioned earlier in this report. The method reduces the double-use of information by splitting the sample of demand observations in half. One half of the sample is used to derive a policy rule while the other half is used to forecast the policy's performance. However, the decision-maker will want to use a policy based on the entire sample of demands. Therefore, we need to understand the relationship between the split-sample forecast and the performance of the policy based on the entire sample. This method is presently under investigation.

## APPENDICES

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APPENDIX A

TABLE A1: SINGLE ITEM FORECASTS OF HOLDING AND BACKLOG QUANTITY USING RETROSPECTIVE SIMULATION

FORECASTS OF HOLDING QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 5.6002 | 5.9907 | -0.39045 | 2.2639 | 6.572 |
| 2 | 0 | 4 | 64 | 6.4698 | 6.6864 | -0.21668 | 3.2001 | 3.349 |
| 2 | 0 | 99 | 32 | 17.1250 | 17.3214 | -0.19647 | 3.0996 | 1.147 |
| 2 | 0 | 99 | 64 | 17.2720 | 17.9968 | -0.72483 | 3.9219 | 4.197 |
| 2 | 4 | 4 | 32 | 10.2225 | 9.9711 | 0.25140 | 5.0215 | 2.459 |
| 2 | 4 | 4 | 64 | 12.1502 | 10.5591 | 1.59107 | 6.4146 | 13.095 |
| 2 | 4 | 99 | 32 | 38.7644 | 38.7345 | 0.02991 | 8.1739 | 0.077 |
| 2 | 4 | 99 | 64 | 34.9989 | 35.9469 | -0.94799 | 8.7170 | 2.709 |
| 8 | 0 | 4 | 32 | 11.2149 | 10.7234 | 0.49158 | 2.4595 | 4.383 |
| 8 | 0 | 4 | 64 | 12.8777 | 12.9210 | -0.04332 | 3.6236 | 0.336 |
| 8 | 0 | 99 | 32 | 31.5095 | 31.9705 | -0.46089 | 3.6828 | 1.463 |
| 8 | 0 | 99 | 64 | 33.2635 | 33.8453 | -0.58183 | 3.9835 | 1.749 |
| 8 | 4 | 4 | 32 | 23.7531 | 19.9626 | 3.79076 | 8.6187 | 15.959 |
| 8 | 4 | 4 | 64 | 22.1750 | 19.8691 | 2.30595 | 8.7471 | 10.399 |
| 8 | 4 | 99 | 32 | 70.7571 | 69.6061 | 1.15128 | 12.3736 | 1.627 |
| 8 | 4 | 99 | 64 | 68.6904 | 67.8507 | 0.84005 | 12.2216 | 1.223 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 416.8440 | 409.9551 | 6.88955 | 27.4116 | 1.653 |

FORECASTS OF BACKLOG QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.7915 | 0.6815 | 0.10993 | 0.6524 | 13.890 |
| 2 | 0 | 4 | 64 | 0.8618 | 0.7469 | 0.11486 | 0.7244 | 13.328 |
| 2 | 0 | 99 | 32 | 0.1678 | 0.0135 | 0.15430 | 0.1523 | 91.976 |
| 2 | 0 | 99 | 64 | 0.1802 | 0.0138 | 0.16633 | 0.1666 | 92.315 |
| 2 | 4 | 4 | 32 | 2.2398 | 0.6742 | 1.56561 | 1.8277 | 69.899 |
| 2 | 4 | 4 | 64 | 2.0322 | 0.9258 | 1.10649 | 1.9808 | 54.447 |
| 2 | 4 | 99 | 32 | 0.2696 | 0.0 | 0.26957 | 0.4920 | 100.000 |
| 2 | 4 | 99 | 64 | 0.3340 | 0.0 | 0.33397 | 0.4649 | 100.000 |
| 8 | 0 | 4 | 32 | 1.6231 | 1.5546 | 0.06851 | 1.2069 | 4.221 |
| 8 | 0 | 4 | 64 | 1.8043 | 1.8469 | -0.04255 | 1.2381 | 2.358 |
| 8 | 0 | 99 | 32 | 0.1980 | 0.0135 | 0.18454 | 0.2151 | 93.201 |
| 8 | 0 | 99 | 64 | 0.2082 | 0.0215 | 0.18668 | 0.2171 | 89.656 |
| 8 | 4 | 4 | 32 | 3.6077 | 1.4319 | 2.17581 | 3.7214 | 60.310 |
| 8 | 4 | 4 | 64 | 4.0402 | 2.2550 | 1.78522 | 3.5572 | 44.187 |
| 8 | 4 | 99 | 32 | 0.3521 | 0.0046 | 0.34752 | 0.6062 | 98.689 |
| 8 | 4 | 99 | 64 | 0.4078 | 0.0 | 0.40782 | 0.7171 | 100.000 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 19.1182 | 10.1837 | 8.93460 | 6.2602 | 46.733 |

TABLE A2: SINGLE ITEM FORECASTS OF BACKLOG AND REPLENISHMENT FREQUENCY USING RETROSPECTIVE SIMULATION

FORECASTS OF BACKLOG FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.1494 | 0.1342 | 0.01514 | 0.1175 | 10.138 |
| 2 | 0 | 4 | 64 | 0.1794 | 0.1788 | 0.00061 | 0.1855 | 0.339 |
| 2 | 0 | 99 | 32 | 0.0239 | 0.0069 | 0.01696 | 0.0316 | 71.009 |
| 2 | 0 | 99 | 64 | 0.0259 | 0.0065 | 0.01940 | 0.0334 | 74.795 |
| 2 | 4 | 4 | 32 | 0.2288 | 0.1431 | 0.08574 | 0.1873 | 37.472 |
| 2 | 4 | 4 | 64 | 0.2073 | 0.1488 | 0.05845 | 0.1921 | 28.198 |
| 2 | 4 | 99 | 32 | 0.0283 | 0.0 | 0.02826 | 0.0507 | 100.000 |
| 2 | 4 | 99 | 64 | 0.0351 | 0.0 | 0.03507 | 0.0481 | 100.000 |
| 8 | 0 | 4 | 32 | 0.1803 | 0.1873 | -0.00696 | 0.0922 | 3.860 |
| 8 | 0 | 4 | 64 | 0.2000 | 0.2135 | -0.01349 | 0.0941 | 6.746 |
| 8 | 0 | 99 | 32 | 0.0220 | 0.0042 | 0.01777 | 0.0291 | 80.769 |
| 8 | 0 | 99 | 64 | 0.0231 | 0.0050 | 0.01814 | 0.0291 | 78.388 |
| 8 | 4 | 4 | 32 | 0.2112 | 0.1527 | 0.05848 | 0.1864 | 27.692 |
| 8 | 4 | 4 | 64 | 0.2341 | 0.2011 | 0.03295 | 0.1671 | 14.074 |
| 8 | 4 | 99 | 32 | 0.0249 | 0.0008 | 0.02409 | 0.0403 | 96.906 |
| 8 | 4 | 99 | 64 | 0.0284 | 0.0 | 0.02838 | 0.0464 | 100.000 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 0.1126 | 0.0864 | 0.02619 | 0.0288 | 23.251 |

FORECASTS OF REPLENISHMENT FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.1185 | 0.1108 | 0.00769 | 0.0904 | 6.494 |
| 2 | 0 | 4 | 64 | 0.0919 | 0.0835 | 0.00846 | 0.0696 | 9.205 |
| 2 | 0 | 99 | 32 | 0.1135 | 0.1108 | 0.00269 | 0.0844 | 2.373 |
| 2 | 0 | 99 | 64 | 0.0915 | 0.0823 | 0.00923 | 0.0779 | 10.084 |
| 2 | 4 | 4 | 32 | 0.0977 | 0.0958 | 0.00192 | 0.0797 | 1.969 |
| 2 | 4 | 4 | 64 | 0.0808 | 0.0773 | 0.00346 | 0.0672 | 4.286 |
| 2 | 4 | 99 | 32 | 0.0819 | 0.0823 | -0.00038 | 0.0694 | 0.469 |
| 2 | 4 | 99 | 64 | 0.0635 | 0.0631 | 0.00038 | 0.0568 | 0.606 |
| 8 | 0 | 4 | 32 | 0.2623 | 0.2519 | 0.01038 | 0.0966 | 3.959 |
| 8 | 0 | 4 | 64 | 0.2008 | 0.1938 | 0.00692 | 0.0774 | 3.448 |
| 8 | 0 | 99 | 32 | 0.2527 | 0.2504 | 0.00231 | 0.0939 | 0.913 |
| 8 | 0 | 99 | 64 | 0.1942 | 0.1942 | 0.0 | 0.0738 | 0.0 |
| 8 | 4 | 4 | 32 | 0.2354 | 0.2288 | 0.00654 | 0.0909 | 2.778 |
| 8 | 4 | 4 | 64 | 0.1777 | 0.1754 | 0.00231 | 0.0696 | 1.299 |
| 8 | 4 | 99 | 32 | 0.2054 | 0.1969 | 0.00846 | 0.0916 | 4.120 |
| 8 | 4 | 99 | 64 | 0.1635 | 0.1546 | 0.00885 | 0.0778 | 5.412 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 0.1519 | 0.1470 | 0.00495 | 0.0200 | 3.259 |

TABLE A3: SINGLE ITEM FORECASTS OF TOTAL COST USING RETROSPECTIVE SIMULATION

FORECASTS OF COST PER PERIOD IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 12.5569 | 12.2614 | 0.29543 | 4.9929 | 2.353 |
| 2 | 0 | 4 | 64 | 15.7999 | 15.0156 | 0.78429 | 6.5791 | 4.964 |
| 2 | 0 | 99 | 32 | 37.3637 | 22.1986 | 15.16576 | 15.5762 | 40.590 |
| 2 | 0 | 99 | 64 | 40.9674 | 24.6349 | 16.33299 | 18.4283 | 39.868 |
| 2 | 4 | 4 | 32 | 22.3079 | 15.7326 | 6.57538 | 7.2005 | 29.476 |
| 2 | 4 | 4 | 64 | 25.4481 | 19.2099 | 6.23854 | 7.9139 | 24.515 |
| 2 | 4 | 99 | 32 | 68.0734 | 41.3682 | 26.70518 | 46.1822 | 39.230 |
| 2 | 4 | 99 | 64 | 72.1237 | 39.9836 | 32.13974 | 44.5899 | 44.562 |
| 8 | 0 | 4 | 32 | 26.1009 | 25.0030 | 1.09791 | 6.1543 | 4.206 |
| 8 | 0 | 4 | 64 | 32.9436 | 32.7142 | 0.22954 | 8.1056 | 0.697 |
| 8 | 0 | 99 | 32 | 59.1978 | 41.3151 | 17.88274 | 21.0649 | 30.208 |
| 8 | 0 | 99 | 64 | 66.3079 | 48.4079 | 17.90004 | 22.2001 | 26.995 |
| 8 | 4 | 4 | 32 | 45.7158 | 33.0126 | 12.70343 | 12.1058 | 27.788 |
| 8 | 4 | 4 | 64 | 49.7073 | 40.1127 | 9.59461 | 11.6281 | 19.302 |
| 8 | 4 | 99 | 32 | 112.1910 | 76.3643 | 35.82608 | 55.7844 | 31.933 |
| 8 | 4 | 99 | 64 | 119.5264 | 77.7456 | 41.77965 | 66.9429 | 34.954 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 806.3311 | 565.0793 | 241.25125 | 117.4782 | 29.920 |

## APPENDIX B

TABLE B1: SINGLE ITEM FORECASTS OF HOLDING AND BACKLOG QUANTITY USING ANALYTICAL APPROXIMATIONS

FORECASTS OF HOLDING QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 5.7559 | 5.5454 | 0.21045 | 2.0967 | 3.656 |
| 2 | 0 | 4 | 64 | 6.3302 | 6.1534 | 0.17672 | 2.7772 | 2.792 |
| 2 | 0 | 99 | 32 | 17.3530 | 14.8840 | 2.46888 | 4.5225 | 14.227 |
| 2 | 0 | 99 | 64 | 17.8625 | 15.6952 | 2.16731 | 4.5382 | 12.133 |
| 2 | 4 | 4 | 32 | 10.5087 | 8.5186 | 1.99010 | 5.4016 | 18.938 |
| 2 | 4 | 4 | 64 | 12.7705 | 9.4274 | 3.34308 | 6.4307 | 26.178 |
| 2 | 4 | 99 | 32 | 38.3212 | 28.9884 | 9.33292 | 11.4743 | 24.354 |
| 2 | 4 | 99 | 64 | 36.3076 | 25.7925 | 10.51562 | 13.0000 | 28.963 |
| 8 | 0 | 4 | 32 | 11.3903 | 11.5292 | -0.13880 | 2.2235 | 1.219 |
| 8 | 0 | 4 | 64 | 12.6214 | 12.6465 | -0.02502 | 2.8436 | 0.198 |
| 8 | 0 | 99 | 32 | 32.1836 | 30.6001 | 1.58363 | 2.7634 | 4.921 |
| 8 | 0 | 99 | 64 | 32.9880 | 31.7882 | 1.19975 | 3.5203 | 3.637 |
| 8 | 4 | 4 | 32 | 23.6489 | 20.9221 | 2.72702 | 7.9766 | 11.531 |
| 8 | 4 | 4 | 64 | 22.7877 | 19.7832 | 3.00465 | 8.3054 | 13.185 |
| 8 | 4 | 99 | 32 | 71.6220 | 65.5094 | 6.11267 | 13.6658 | 8.535 |
| 8 | 4 | 99 | 64 | 69.0973 | 63.3080 | 5.78939 | 13.7254 | 8.379 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 421.5483 | 371.0913 | 50.45830 | 31.0669 | 11.970 |

FORECASTS OF BACKLOG QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.7741 | 1.0287 | -0.25460 | 0.7938 | 32.892 |
| 2 | 0 | 4 | 64 | 0.9083 | 1.0140 | -0.10567 | 0.8832 | 11.633 |
| 2 | 0 | 99 | 32 | 0.1693 | 0.0591 | 0.11021 | 0.1781 | 65.110 |
| 2 | 0 | 99 | 64 | 0.1654 | 0.0574 | 0.10798 | 0.1613 | 65.303 |
| 2 | 4 | 4 | 32 | 2.2826 | 1.1231 | 1.15954 | 2.0667 | 50.799 |
| 2 | 4 | 4 | 64 | 1.9233 | 1.4043 | 0.51898 | 1.8996 | 26.984 |
| 2 | 4 | 99 | 32 | 0.2563 | 0.0650 | 0.19128 | 0.4523 | 74.638 |
| 2 | 4 | 99 | 64 | 0.3000 | 0.0637 | 0.23630 | 0.4538 | 78.771 |
| 8 | 0 | 4 | 32 | 1.5735 | 1.7307 | -0.15718 | 1.1850 | 9.989 |
| 8 | 0 | 4 | 64 | 1.9292 | 1.7802 | 0.14903 | 1.3465 | 7.725 |
| 8 | 0 | 99 | 32 | 0.1795 | 0.0824 | 0.09707 | 0.1980 | 54.090 |
| 8 | 0 | 99 | 64 | 0.2100 | 0.0790 | 0.13102 | 0.2287 | 62.386 |
| 8 | 4 | 4 | 32 | 3.3462 | 2.2899 | 1.05628 | 3.4010 | 31.566 |
| 8 | 4 | 4 | 64 | 4.1545 | 2.4202 | 1.73438 | 3.9330 | 41.747 |
| 8 | 4 | 99 | 32 | 0.3394 | 0.1006 | 0.23881 | 0.6427 | 70.365 |
| 8 | 4 | 99 | 64 | 0.4062 | 0.1076 | 0.29855 | 0.7277 | 73.503 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 18.9176 | 13.4057 | 5.51198 | 6.4066 | 29.137 |

TABLE B2: SINGLE ITEM FORECASTS OF BACKLOG AND REPLENISHMENT FREQUENCY USING ANALYTICAL APPROXIMATIONS

FORECASTS OF BACKLOG FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.1450 | 0.1880 | -0.04297 | 0.0890 | 29.627 |
| 2 | 0 | 4 | 64 | 0.1928 | 0.2266 | -0.03379 | 0.1535 | 17.526 |
| 2 | 0 | 99 | 32 | 0.0242 | 0.0351 | -0.01093 | 0.0319 | 45.205 |
| 2 | 0 | 99 | 64 | 0.0237 | 0.0258 | -0.00208 | 0.0278 | 8.780 |
| 2 | 4 | 4 | 32 | 0.2326 | 0.1274 | 0.10516 | 0.1316 | 45.218 |
| 2 | 4 | 4 | 64 | 0.1961 | 0.1766 | 0.01949 | 0.1326 | 9.940 |
| 2 | 4 | 99 | 32 | 0.0269 | 0.0475 | -0.02058 | 0.0470 | 76.535 |
| 2 | 4 | 99 | 64 | 0.0315 | 0.0457 | -0.01422 | 0.0484 | 45.104 |
| 8 | 0 | 4 | 32 | 0.1748 | 0.1428 | 0.03208 | 0.0563 | 18.350 |
| 8 | 0 | 4 | 64 | 0.2138 | 0.2004 | 0.01348 | 0.0829 | 6.304 |
| 8 | 0 | 99 | 32 | 0.0199 | 0.0420 | -0.02202 | 0.0249 | 110.431 |
| 8 | 0 | 99 | 64 | 0.0233 | 0.0347 | -0.01141 | 0.0281 | 48.900 |
| 8 | 4 | 4 | 32 | 0.1996 | 0.1342 | 0.06539 | 0.1354 | 32.762 |
| 8 | 4 | 4 | 64 | 0.2356 | 0.1743 | 0.06137 | 0.1416 | 26.045 |
| 8 | 4 | 99 | 32 | 0.0239 | 0.0170 | 0.00688 | 0.0397 | 28.841 |
| 8 | 4 | 99 | 64 | 0.0281 | 0.0228 | 0.00538 | 0.0458 | 19.117 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 0.1120 | 0.1025 | 0.00945 | 0.0222 | 8.441 |

FORECASTS OF REPLENISHMENT FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.1235 | 0.1177 | 0.00577 | 0.0820 | 4.674 |
| 2 | 0 | 4 | 64 | 0.0954 | 0.0867 | 0.00870 | 0.0719 | 9.121 |
| 2 | 0 | 99 | 32 | 0.1115 | 0.1164 | -0.00488 | 0.0817 | 4.374 |
| 2 | 0 | 99 | 64 | 0.0919 | 0.0901 | 0.00177 | 0.0661 | 1.931 |
| 2 | 4 | 4 | 32 | 0.0942 | 0.1015 | -0.00730 | 0.0651 | 7.742 |
| 2 | 4 | 4 | 64 | 0.0815 | 0.0814 | 0.00010 | 0.0609 | 0.119 |
| 2 | 4 | 99 | 32 | 0.0804 | 0.1084 | -0.02800 | 0.0660 | 34.827 |
| 2 | 4 | 99 | 64 | 0.0615 | 0.0773 | -0.01579 | 0.0576 | 25.663 |
| 8 | 0 | 4 | 32 | 0.2596 | 0.2600 | -0.00036 | 0.0805 | 0.137 |
| 8 | 0 | 4 | 64 | 0.1973 | 0.1966 | 0.00071 | 0.0784 | 0.358 |
| 8 | 0 | 99 | 32 | 0.2558 | 0.2578 | -0.00199 | 0.0810 | 0.777 |
| 8 | 0 | 99 | 64 | 0.1946 | 0.1952 | -0.00060 | 0.0710 | 0.309 |
| 8 | 4 | 4 | 32 | 0.2350 | 0.2409 | -0.00590 | 0.0808 | 2.512 |
| 8 | 4 | 4 | 64 | 0.1758 | 0.1801 | -0.00431 | 0.0690 | 2.449 |
| 8 | 4 | 99 | 32 | 0.2042 | 0.2338 | -0.02954 | 0.1008 | 14.466 |
| 8 | 4 | 99 | 64 | 0.1581 | 0.1763 | -0.01823 | 0.0733 | 11.533 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 0.1513 | 0.1575 | -0.00624 | 0.0187 | 4.125 |

TABLE B3: SINGLE ITEM FORECASTS OF TOTAL COST USING ANALYTICAL APPROXIMATIONS

FORECASTS OF COST PER PERIOD IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 12.8029 | 13.4262 | -0.62334 | 5.2738 | 4.869 |
| 2 | 0 | 4 | 64 | 16.0681 | 15.7573 | 0.31086 | 7.3210 | 1.935 |
| 2 | 0 | 99 | 32 | 37.6790 | 24.4558 | 13.22353 | 18.2281 | 35.095 |
| 2 | 0 | 99 | 64 | 40.1153 | 27.1444 | 12.97130 | 17.3346 | 32.335 |
| 2 | 4 | 4 | 32 | 22.6545 | 16.2598 | 6.39476 | 7.9664 | 28.227 |
| 2 | 4 | 4 | 64 | 25.6819 | 20.2569 | 5.42522 | 8.0510 | 21.125 |
| 2 | 4 | 99 | 32 | 66.2654 | 38.8910 | 27.37402 | 43.5107 | 41.310 |
| 2 | 4 | 99 | 64 | 69.9445 | 37.0461 | 32.89818 | 42.3456 | 47.035 |
| 8 | 0 | 4 | 32 | 25.9917 | 26.7707 | -0.77897 | 6.2928 | 2.597 |
| 8 | 0 | 4 | 64 | 32.9654 | 32.3492 | 0.61627 | 7.9287 | 1.869 |
| 8 | 0 | 99 | 32 | 58.1348 | 47.0048 | 11.13026 | 19.8089 | 19.146 |
| 8 | 0 | 99 | 64 | 66.2348 | 52.1022 | 14.13248 | 22.7541 | 21.337 |
| 8 | 4 | 4 | 32 | 44.5533 | 37.7902 | 6.76325 | 11.0078 | 15.180 |
| 8 | 4 | 4 | 64 | 50.6546 | 40.9879 | 9.66674 | 13.6830 | 19.084 |
| 8 | 4 | 99 | 32 | 111.7568 | 82.9472 | 28.80920 | 59.4302 | 25.778 |
| 8 | 4 | 99 | 64 | 119.4257 | 85.2462 | 34.17871 | 67.1747 | 28.619 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 800.9277 | 598.4351 | 202.49242 | 117.8735 | 25.282 |

## APPENDIX C

TABLE C1: SINGLE ITEM FORECASTS OF HOLDING AND BACKLOG QUANTITY USING FIXED POLICY MULTIPLICATIVE APPROXIMATIONS

FORECASTS OF HOLDING QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 5.7559 | 5.5773 | 0.17862 | 2.2125 | 3.103 |
| 2 | 0 | 4 | 64 | 6.3302 | 5.4444 | 0.88571 | 2.6984 | 13.992 |
| 2 | 0 | 99 | 32 | 17.3530 | 17.4110 | -0.05797 | 6.9868 | 0.334 |
| 2 | 0 | 99 | 64 | 17.8625 | 16.9782 | 0.88437 | 5.6403 | 4.951 |
| 2 | 4 | 4 | 32 | 10.5087 | 8.3657 | 2.14300 | 5.6455 | 20.393 |
| 2 | 4 | 4 | 64 | 12.7705 | 9.7645 | 3.00595 | 6.4004 | 23.538 |
| 2 | 4 | 99 | 32 | 38.3212 | 30.4619 | 7.85931 | 16.6573 | 20.509 |
| 2 | 4 | 99 | 64 | 36.3076 | 26.8648 | 9.44326 | 14.5717 | 26.009 |
| 8 | 0 | 4 | 32 | 11.3903 | 11.6792 | -0.28884 | 2.5709 | 2.536 |
| 8 | 0 | 4 | 64 | 12.6214 | 11.8176 | 0.80386 | 2.8494 | 6.369 |
| 8 | 0 | 99 | 32 | 32.1836 | 35.7039 | -3.52029 | 9.5785 | 10.938 |
| 8 | 0 | 99 | 64 | 32.9880 | 34.7650 | -1.77701 | 6.8766 | 5.387 |
| 8 | 4 | 4 | 32 | 23.6489 | 20.6733 | 2.97582 | 8.5521 | 12.583 |
| 8 | 4 | 4 | 64 | 22.7877 | 20.6147 | 2.17315 | 8.3902 | 9.537 |
| 8 | 4 | 99 | 32 | 71.6220 | 69.4931 | 2.12900 | 39.7019 | 2.973 |
| 8 | 4 | 99 | 64 | 69.0973 | 66.8083 | 2.28910 | 23.5112 | 3.313 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 421.5483 | 392.4224 | 29.12698 | 55.5739 | 6.910 |

FORECASTS OF BACKLOG QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.7741 | 0.7620 | 0.01210 | 0.5532 | 1.563 |
| 2 | 0 | 4 | 64 | 0.9083 | 0.6591 | 0.24920 | 0.6200 | 27.435 |
| 2 | 0 | 99 | 32 | 0.1693 | 0.0400 | 0.12929 | 0.1670 | 76.381 |
| 2 | 0 | 99 | 64 | 0.1654 | 0.0500 | 0.11531 | 0.1588 | 69.735 |
| 2 | 4 | 4 | 32 | 2.2826 | 1.1294 | 1.15324 | 1.8906 | 50.523 |
| 2 | 4 | 4 | 64 | 1.9233 | 1.4173 | 0.50598 | 1.7667 | 26.308 |
| 2 | 4 | 99 | 32 | 0.2563 | 0.0334 | 0.22288 | 0.4403 | 86.969 |
| 2 | 4 | 99 | 64 | 0.3000 | 0.0581 | 0.24188 | 0.4543 | 80.633 |
| 8 | 0 | 4 | 32 | 1.5735 | 1.1802 | 0.39335 | 0.9410 | 24.998 |
| 8 | 0 | 4 | 64 | 1.9292 | 0.9398 | 0.98945 | 0.9975 | 51.288 |
| 8 | 0 | 99 | 32 | 0.1795 | 0.0657 | 0.11375 | 0.1819 | 63.385 |
| 8 | 0 | 99 | 64 | 0.2100 | 0.0818 | 0.12819 | 0.2231 | 61.040 |
| 8 | 4 | 4 | 32 | 3.3462 | 2.2334 | 1.11281 | 3.0942 | 33.256 |
| 8 | 4 | 4 | 64 | 4.1545 | 2.4835 | 1.67109 | 3.4777 | 40.223 |
| 8 | 4 | 99 | 32 | 0.3394 | 0.0217 | 0.31771 | 0.6512 | 93.615 |
| 8 | 4 | 99 | 64 | 0.4062 | 0.1008 | 0.30542 | 0.7254 | 75.194 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 18.9176 | 11.2561 | 7.66165 | 5.6940 | 40.500 |

TABLE C2: SINGLE ITEM FORECASTS OF BACKLOG AND REPLENISHMENT FREQUENCY USING FIXED POLICY MULTIPLICATIVE APPROXIMATIONS

FORECASTS OF BACKLOG FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.1450 | 0.1889 | -0.04386 | 0.0966 | 30.241 |
| 2 | 0 | 4 | 64 | 0.1928 | 0.1889 | 0.00392 | 0.1527 | 2.033 |
| 2 | 0 | 99 | 32 | 0.0242 | 0.0090 | 0.01515 | 0.0235 | 62.677 |
| 2 | 0 | 99 | 64 | 0.0237 | 0.0090 | 0.01467 | 0.0211 | 61.923 |
| 2 | 4 | 4 | 32 | 0.2326 | 0.1889 | 0.04368 | 0.1561 | 18.783 |
| 2 | 4 | 4 | 64 | 0.1961 | 0.1889 | 0.00724 | 0.1408 | 3.694 |
| 2 | 4 | 99 | 32 | 0.0269 | 0.0090 | 0.01786 | 0.0444 | 66.439 |
| 2 | 4 | 99 | 64 | 0.0315 | 0.0090 | 0.02254 | 0.0453 | 71.516 |
| 8 | 0 | 4 | 32 | 0.1748 | 0.1889 | -0.01405 | 0.0715 | 8.034 |
| 8 | 0 | 4 | 64 | 0.2138 | 0.1889 | 0.02496 | 0.0925 | 11.674 |
| 8 | 0 | 99 | 32 | 0.0199 | 0.0090 | 0.01092 | 0.0189 | 54.751 |
| 8 | 0 | 99 | 64 | 0.0233 | 0.0090 | 0.01431 | 0.0225 | 61.334 |
| 8 | 4 | 4 | 32 | 0.1996 | 0.1889 | 0.01072 | 0.1457 | 5.372 |
| 8 | 4 | 4 | 64 | 0.2356 | 0.1889 | 0.04675 | 0.1575 | 19.839 |
| 8 | 4 | 99 | 32 | 0.0239 | 0.0090 | 0.01483 | 0.0408 | 62.170 |
| 8 | 4 | 99 | 64 | 0.0281 | 0.0090 | 0.01912 | 0.0454 | 67.944 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 0.1120 | 0.0989 | 0.01305 | 0.0239 | 11.651 |

FORECASTS OF REPLENISHMENT FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.1235 | 0.1213 | 0.00219 | 0.0821 | 1.770 |
| 2 | 0 | 4 | 64 | 0.0954 | 0.0888 | 0.00654 | 0.0718 | 6.860 |
| 2 | 0 | 99 | 32 | 0.1115 | 0.1204 | -0.00889 | 0.0807 | 7.970 |
| 2 | 0 | 99 | 64 | 0.0919 | 0.0918 | 0.00014 | 0.0659 | 0.154 |
| 2 | 4 | 4 | 32 | 0.0942 | 0.1040 | -0.00977 | 0.0647 | 10.368 |
| 2 | 4 | 4 | 64 | 0.0815 | 0.0834 | -0.00185 | 0.0608 | 2.270 |
| 2 | 4 | 99 | 32 | 0.0804 | 0.1107 | -0.03029 | 0.0658 | 37.685 |
| 2 | 4 | 99 | 64 | 0.0615 | 0.0794 | -0.01786 | 0.0576 | 29.028 |
| 8 | 0 | 4 | 32 | 0.2596 | 0.2633 | -0.00367 | 0.0842 | 1.413 |
| 8 | 0 | 4 | 64 | 0.1973 | 0.1971 | 0.00022 | 0.0792 | 0.109 |
| 8 | 0 | 99 | 32 | 0.2558 | 0.2598 | -0.00403 | 0.0822 | 1.574 |
| 8 | 0 | 99 | 64 | 0.1946 | 0.1943 | 0.00036 | 0.0711 | 0.185 |
| 8 | 4 | 4 | 32 | 0.2350 | 0.2410 | -0.00600 | 0.0803 | 2.553 |
| 8 | 4 | 4 | 64 | 0.1758 | 0.1794 | -0.00366 | 0.0702 | 2.080 |
| 8 | 4 | 99 | 32 | 0.2042 | 0.2351 | -0.03086 | 0.0996 | 15.109 |
| 8 | 4 | 99 | 64 | 0.1581 | 0.1763 | -0.01827 | 0.0730 | 11.556 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 0.1513 | 0.1591 | -0.00786 | 0.0188 | 5.193 |

TABLE C3: SINGLE ITEM FORECASTS OF TOTAL COST USING FIXED POLICY MULTIPLICATIVE APPROXIMATIONS

FORECASTS OF COST PER PERIOD IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 12.8029 | 12.5059 | 0.29694 | 4.1409 | 2.319 |
| 2 | 0 | 4 | 64 | 16.0681 | 13.7668 | 2.30128 | 5.8724 | 14.322 |
| 2 | 0 | 99 | 32 | 37.6790 | 25.2223 | 12.45695 | 19.7112 | 33.061 |
| 2 | 0 | 99 | 64 | 40.1153 | 27.8063 | 12.30938 | 18.5789 | 30.685 |
| 2 | 4 | 4 | 32 | 22.6545 | 16.2113 | 6.44329 | 6.9392 | 28.441 |
| 2 | 4 | 4 | 64 | 25.6819 | 20.7707 | 4.91142 | 7.4228 | 19.124 |
| 2 | 4 | 99 | 32 | 66.2654 | 37.3097 | 28.95544 | 44.7346 | 43.696 |
| 2 | 4 | 99 | 64 | 69.9445 | 37.6980 | 32.24626 | 44.3359 | 46.103 |
| 8 | 0 | 4 | 32 | 25.9917 | 24.8247 | 1.16719 | 3.9230 | 4.491 |
| 8 | 0 | 4 | 64 | 32.9654 | 28.1901 | 4.77534 | 6.0677 | 14.486 |
| 8 | 0 | 99 | 32 | 58.1348 | 50.5224 | 7.61253 | 22.1837 | 13.095 |
| 8 | 0 | 99 | 64 | 66.2348 | 55.2975 | 10.93728 | 24.4578 | 16.513 |
| 8 | 4 | 4 | 32 | 44.5533 | 37.3183 | 7.23502 | 8.8645 | 16.239 |
| 8 | 4 | 4 | 64 | 50.6546 | 42.0312 | 8.62354 | 11.7686 | 17.024 |
| 8 | 4 | 99 | 32 | 111.7568 | 79.1610 | 32.59503 | 62.0410 | 29.166 |
| 8 | 4 | 99 | 64 | 119.4257 | 88.0688 | 31.35599 | 70.6379 | 26.256 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 800.9277 | 596.7041 | 204.22289 | 122.6940 | 25.498 |

## APPENDIX D

TABLE D1: SINGLE ITEM FORECASTS OF HOLDING AND BACKLOG QUANTITY USING STATISTICAL POLICY MULTIPLICATIVE APPROXIMATIONS

FORECASTS OF HOLDING QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 5.7559 | 5.6788 | 0.07704 | 2.1358 | 1.338 |
| 2 | 0 | 4 | 64 | 6.3302 | 5.8169 | 0.51323 | 2.6508 | 8.108 |
| 2 | 0 | 99 | 32 | 17.3530 | 17.1024 | 0.25055 | 5.3486 | 1.444 |
| 2 | 0 | 99 | 64 | 17.8625 | 18.2989 | -0.43634 | 5.2488 | 2.443 |
| 2 | 4 | 4 | 32 | 10.5087 | 10.2823 | 0.22646 | 6.4027 | 2.155 |
| 2 | 4 | 4 | 64 | 12.7705 | 11.8109 | 0.95963 | 6.7264 | 7.514 |
| 2 | 4 | 99 | 32 | 38.3212 | 33.6644 | 4.65674 | 14.3411 | 12.152 |
| 2 | 4 | 99 | 64 | 36.3076 | 33.9444 | 2.36339 | 14.5234 | 6.509 |
| 8 | 0 | 4 | 32 | 11.3903 | 10.9809 | 0.40946 | 2.6335 | 3.595 |
| 8 | 0 | 4 | 64 | 12.6214 | 11.6424 | 0.97903 | 3.2144 | 7.757 |
| 8 | 0 | 99 | 32 | 32.1836 | 32.7755 | -0.59198 | 7.2200 | 1.839 |
| 8 | 0 | 99 | 64 | 32.9880 | 34.6410 | -1.65285 | 7.3618 | 5.010 |
| 8 | 4 | 4 | 32 | 23.6489 | 21.5945 | 2.05455 | 9.9554 | 8.688 |
| 8 | 4 | 4 | 64 | 22.7877 | 22.8054 | -0.01766 | 10.4080 | 0.077 |
| 8 | 4 | 99 | 32 | 71.6220 | 64.0373 | 7.58482 | 24.7646 | 10.590 |
| 8 | 4 | 99 | 64 | 69.0973 | 68.2749 | 0.82252 | 23.0494 | 1.190 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 421.5483 | 403.3503 | 18.19858 | 45.2346 | 4.317 |

FORECASTS OF BACKLOG QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.7741 | 1.1072 | -0.33317 | 0.4871 | 43.043 |
| 2 | 0 | 4 | 64 | 0.9083 | 0.9283 | -0.02001 | 0.5858 | 2.203 |
| 2 | 0 | 99 | 32 | 0.1693 | 0.1221 | 0.04717 | 0.1747 | 27.869 |
| 2 | 0 | 99 | 64 | 0.1654 | 0.1335 | 0.03187 | 0.1580 | 19.272 |
| 2 | 4 | 4 | 32 | 2.2826 | 2.1841 | 0.09853 | 1.9808 | 4.316 |
| 2 | 4 | 4 | 64 | 1.9233 | 2.4783 | -0.55508 | 1.7934 | 28.861 |
| 2 | 4 | 99 | 32 | 0.2563 | 0.1991 | 0.05722 | 0.4607 | 22.328 |
| 2 | 4 | 99 | 64 | 0.3000 | 0.2211 | 0.07885 | 0.4669 | 26.286 |
| 8 | 0 | 4 | 32 | 1.5735 | 1.7196 | -0.14607 | 0.7381 | 9.283 |
| 8 | 0 | 4 | 64 | 1.9292 | 1.2268 | 0.70243 | 0.8535 | 36.410 |
| 8 | 0 | 99 | 32 | 0.1795 | 0.2119 | -0.03247 | 0.1880 | 18.092 |
| 8 | 0 | 99 | 64 | 0.2100 | 0.2232 | -0.01322 | 0.2194 | 6.296 |
| 8 | 4 | 4 | 32 | 3.3462 | 4.0898 | -0.74364 | 3.2055 | 22.223 |
| 8 | 4 | 4 | 64 | 4.1545 | 4.1749 | -0.02034 | 3.7115 | 0.490 |
| 8 | 4 | 99 | 32 | 0.3394 | 0.3518 | -0.01239 | 0.6479 | 3.650 |
| 8 | 4 | 99 | 64 | 0.4062 | 0.4079 | -0.00172 | 0.7364 | 0.424 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 18.9176 | 19.7797 | -0.86204 | 5.8800 | 4.557 |

TABLE D2: SINGLE ITEM FORECASTS OF BACKLOG AND REPLENISHMENT FREQUENCY USING STATISTICAL POLICY MULTIPLICATIVE APPROXIMATIONS

FORECASTS OF BACKLOG FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.1450 | 0.2129 | -0.06787 | 0.0966 | 46.802 |
| 2 | 0 | 4 | 64 | 0.1928 | 0.2129 | -0.02010 | 0.1527 | 10.424 |
| 2 | 0 | 99 | 32 | 0.0242 | 0.0246 | -0.00044 | 0.0235 | 1.800 |
| 2 | 0 | 99 | 64 | 0.0237 | 0.0246 | -0.00091 | 0.0211 | 3.856 |
| 2 | 4 | 4 | 32 | 0.2326 | 0.2129 | 0.01966 | 0.1561 | 8.456 |
| 2 | 4 | 4 | 64 | 0.1961 | 0.2129 | -0.01677 | 0.1408 | 8.552 |
| 2 | 4 | 99 | 32 | 0.0269 | 0.0246 | 0.00228 | 0.0444 | 8.463 |
| 2 | 4 | 99 | 64 | 0.0315 | 0.0245 | 0.00703 | 0.0453 | 22.309 |
| 8 | 0 | 4 | 32 | 0.1748 | 0.2129 | -0.03806 | 0.0715 | 21.771 |
| 8 | 0 | 4 | 64 | 0.2138 | 0.2129 | 0.00095 | 0.0925 | 0.443 |
| 8 | 0 | 99 | 32 | 0.0199 | 0.0246 | -0.00467 | 0.0189 | 23.418 |
| 8 | 0 | 99 | 64 | 0.0233 | 0.0246 | -0.00127 | 0.0225 | 5.463 |
| 8 | 4 | 4 | 32 | 0.1996 | 0.2129 | -0.01330 | 0.1457 | 6.661 |
| 8 | 4 | 4 | 64 | 0.2356 | 0.2129 | 0.02273 | 0.1575 | 9.646 |
| 8 | 4 | 99 | 32 | 0.0239 | 0.0246 | -0.00076 | 0.0408 | 3.182 |
| 8 | 4 | 99 | 64 | 0.0281 | 0.0246 | 0.00354 | 0.0454 | 12.567 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 0.1120 | 0.1187 | -0.00675 | 0.0239 | 6.025 |

FORECASTS OF REPLENISHMENT FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.1235 | 0.1140 | 0.00949 | 0.0864 | 7.683 |
| 2 | 0 | 4 | 64 | 0.0954 | 0.0843 | 0.01112 | 0.0762 | 11.658 |
| 2 | 0 | 99 | 32 | 0.1115 | 0.1134 | -0.00188 | 0.0822 | 1.688 |
| 2 | 0 | 99 | 64 | 0.0919 | 0.0883 | 0.00366 | 0.0677 | 3.979 |
| 2 | 4 | 4 | 32 | 0.0942 | 0.0867 | 0.00755 | 0.0658 | 8.018 |
| 2 | 4 | 4 | 64 | 0.0815 | 0.0738 | 0.00776 | 0.0603 | 9.522 |
| 2 | 4 | 99 | 32 | 0.0804 | 0.0955 | -0.01510 | 0.0653 | 18.788 |
| 2 | 4 | 99 | 64 | 0.0615 | 0.0694 | -0.00782 | 0.0564 | 12.711 |
| 8 | 0 | 4 | 32 | 0.2596 | 0.2601 | -0.00045 | 0.0883 | 0.172 |
| 8 | 0 | 4 | 64 | 0.1973 | 0.2005 | -0.00316 | 0.0831 | 1.604 |
| 8 | 0 | 99 | 32 | 0.2558 | 0.2555 | 0.00026 | 0.0856 | 0.100 |
| 8 | 0 | 99 | 64 | 0.1946 | 0.1961 | -0.00145 | 0.0726 | 0.746 |
| 8 | 4 | 4 | 32 | 0.2350 | 0.2191 | 0.01588 | 0.0793 | 6.759 |
| 8 | 4 | 4 | 64 | 0.1758 | 0.1681 | 0.00765 | 0.0706 | 4.351 |
| 8 | 4 | 99 | 32 | 0.2042 | 0.2135 | -0.00931 | 0.0967 | 4.560 |
| 8 | 4 | 99 | 64 | 0.1581 | 0.1657 | -0.00762 | 0.0713 | 4.819 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 0.1513 | 0.1502 | 0.00104 | 0.0191 | 0.685 |

TABLE D3: SINGLE ITEM FORECASTS OF TOTAL COST USING STATISTICAL POLICY MULTIPLICATIVE APPROXIMATIONS

FORECASTS OF COST PER PERIOD IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 12.8029 | 13.7550 | -0.95213 | 3.7940 | 7.437 |
| 2 | 0 | 4 | 64 | 16.0681 | 14.9232 | 1.14490 | 5.7769 | 7.125 |
| 2 | 0 | 99 | 32 | 37.6790 | 32.8188 | 4.86028 | 16.4559 | 12.899 |
| 2 | 0 | 99 | 64 | 40.1153 | 37.1629 | 2.95261 | 16.0914 | 7.360 |
| 2 | 4 | 4 | 32 | 22.6545 | 21.7922 | 0.86235 | 6.3744 | 3.807 |
| 2 | 4 | 4 | 64 | 25.6819 | 26.4457 | -0.76383 | 6.9708 | 2.974 |
| 2 | 4 | 99 | 32 | 66.2654 | 56.4266 | 9.83840 | 41.7071 | 14.847 |
| 2 | 4 | 99 | 64 | 69.9445 | 60.2751 | 9.66928 | 42.1199 | 13.824 |
| 8 | 0 | 4 | 32 | 25.9917 | 26.1810 | -0.18912 | 3.5938 | 0.728 |
| 8 | 0 | 4 | 64 | 32.9654 | 29.3793 | 3.58614 | 5.8048 | 10.878 |
| 8 | 0 | 99 | 32 | 58.1348 | 61.9332 | -3.79816 | 15.9836 | 6.533 |
| 8 | 0 | 99 | 64 | 66.2348 | 69.2897 | -3.05477 | 19.8692 | 4.612 |
| 8 | 4 | 4 | 32 | 44.5533 | 44.9652 | -0.41174 | 7.9433 | 0.924 |
| 8 | 4 | 4 | 64 | 50.6546 | 50.2642 | 0.39043 | 11.2720 | 0.771 |
| 8 | 4 | 99 | 32 | 111.7568 | 105.6966 | 6.06043 | 53.6820 | 5.423 |
| 8 | 4 | 99 | 64 | 119.4257 | 119.2611 | 0.16459 | 62.4974 | 0.138 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 800.9277 | 770.5688 | 30.35957 | 108.8806 | 3.791 |

## APPENDIX E

TABLE E1: SINGLE ITEM FORECASTS OF HOLDING AND BACKLOG QUANTITY USING FUTURE DEMAND GENERATION (NEGATIVE BINOMIAL)

FORECASTS OF HOLDING QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 5.6002 | 5.6413 | -0.04112 | 2.1186 | 0.734 |
| 2 | 0 | 4 | 64 | 6.4698 | 6.5008 | -0.03110 | 2.5420 | 0.481 |
| 2 | 0 | 99 | 32 | 17.1250 | 17.2047 | -0.07974 | 2.6047 | 0.466 |
| 2 | 0 | 99 | 64 | 17.2720 | 17.8211 | -0.54910 | 3.4539 | 3.179 |
| 2 | 4 | 4 | 32 | 10.2225 | 10.4106 | -0.18811 | 5.4354 | 1.840 |
| 2 | 4 | 4 | 64 | 12.1502 | 11.4993 | 0.65087 | 6.2964 | 5.357 |
| 2 | 4 | 99 | 32 | 38.7644 | 39.9697 | -1.20554 | 9.4884 | 3.110 |
| 2 | 4 | 99 | 64 | 34.9989 | 36.5015 | -1.50269 | 8.3503 | 4.294 |
| 8 | 0 | 4 | 32 | 11.2149 | 10.8057 | 0.40927 | 1.9508 | 3.649 |
| 8 | 0 | 4 | 64 | 12.8777 | 12.4983 | 0.37937 | 2.5126 | 2.946 |
| 8 | 0 | 99 | 32 | 31.5095 | 31.6278 | -0.11827 | 2.9722 | 0.375 |
| 8 | 0 | 99 | 64 | 33.2635 | 33.6685 | -0.40508 | 3.5754 | 1.218 |
| 8 | 4 | 4 | 32 | 23.7531 | 21.8143 | 1.93897 | 10.6034 | 8.163 |
| 8 | 4 | 4 | 64 | 22.1750 | 20.5423 | 1.63281 | 6.7839 | 7.363 |
| 8 | 4 | 99 | 32 | 70.7571 | 71.3073 | -0.55024 | 13.6889 | 0.778 |
| 8 | 4 | 99 | 64 | 68.6904 | 69.3022 | -0.61172 | 14.6271 | 0.891 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 416.8440 | 417.1147 | -0.27141 | 29.6862 | 0.065 |

FORECASTS OF BACKLOG QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.7915 | 0.6711 | 0.12041 | 0.6144 | 15.214 |
| 2 | 0 | 4 | 64 | 0.8618 | 0.7093 | 0.15250 | 0.6533 | 17.696 |
| 2 | 0 | 99 | 32 | 0.1678 | 0.0603 | 0.10748 | 0.1918 | 64.064 |
| 2 | 0 | 99 | 64 | 0.1802 | 0.0517 | 0.12850 | 0.1907 | 71.316 |
| 2 | 4 | 4 | 32 | 2.2398 | 0.9914 | 1.24842 | 2.0390 | 55.738 |
| 2 | 4 | 4 | 64 | 2.0322 | 1.2637 | 0.76856 | 2.1021 | 37.819 |
| 2 | 4 | 99 | 32 | 0.2696 | 0.0209 | 0.24871 | 0.5020 | 92.260 |
| 2 | 4 | 99 | 64 | 0.3340 | 0.0254 | 0.30854 | 0.4654 | 92.385 |
| 8 | 0 | 4 | 32 | 1.6231 | 1.5603 | 0.06278 | 1.1676 | 3.868 |
| 8 | 0 | 4 | 64 | 1.8043 | 1.8041 | 0.00019 | 1.1163 | 0.010 |
| 8 | 0 | 99 | 32 | 0.1980 | 0.0905 | 0.10752 | 0.2500 | 54.304 |
| 8 | 0 | 99 | 64 | 0.2082 | 0.0911 | 0.11716 | 0.2540 | 56.269 |
| 8 | 4 | 4 | 32 | 3.6077 | 2.3110 | 1.29667 | 4.0301 | 35.942 |
| 8 | 4 | 4 | 64 | 4.0402 | 2.9266 | 1.11357 | 3.6636 | 27.562 |
| 8 | 4 | 99 | 32 | 0.3521 | 0.0386 | 0.31358 | 0.6265 | 89.050 |
| 8 | 4 | 99 | 64 | 0.4078 | 0.0570 | 0.35085 | 0.7472 | 86.030 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 19.1182 | 12.6729 | 6.44542 | 6.5783 | 33.714 |

TABLE E2: SINGLE ITEM FORECASTS OF BACKLOG AND REPLENISHMENT FREQUENCY USING FUTURE DEMAND GENERATION (NEGATIVE BINOMIAL)

FORECASTS OF BACKLOG FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.1494 | 0.1511 | -0.00168 | 0.0961 | 1.128 |
| 2 | 0 | 4 | 64 | 0.1794 | 0.1754 | 0.00402 | 0.1411 | 2.241 |
| 2 | 0 | 99 | 32 | 0.0239 | 0.0102 | 0.01369 | 0.0270 | 57.319 |
| 2 | 0 | 99 | 64 | 0.0259 | 0.0099 | 0.01609 | 0.0264 | 62.007 |
| 2 | 4 | 4 | 32 | 0.2288 | 0.1371 | 0.09170 | 0.1720 | 40.077 |
| 2 | 4 | 4 | 64 | 0.2073 | 0.1615 | 0.04576 | 0.1692 | 22.075 |
| 2 | 4 | 99 | 32 | 0.0283 | 0.0033 | 0.02499 | 0.0516 | 88.432 |
| 2 | 4 | 99 | 64 | 0.0351 | 0.0040 | 0.03103 | 0.0477 | 88.485 |
| 8 | 0 | 4 | 32 | 0.1803 | 0.1789 | 0.00140 | 0.0767 | 0.779 |
| 8 | 0 | 4 | 64 | 0.2000 | 0.2061 | -0.00609 | 0.0817 | 3.043 |
| 8 | 0 | 99 | 32 | 0.0220 | 0.0113 | 0.01070 | 0.0244 | 48.646 |
| 8 | 0 | 99 | 64 | 0.0231 | 0.0108 | 0.01232 | 0.0242 | 53.244 |
| 8 | 4 | 4 | 32 | 0.2112 | 0.1632 | 0.04799 | 0.1786 | 22.729 |
| 8 | 4 | 4 | 64 | 0.2341 | 0.1985 | 0.03564 | 0.1514 | 15.224 |
| 8 | 4 | 99 | 32 | 0.0249 | 0.0035 | 0.02140 | 0.0412 | 86.077 |
| 8 | 4 | 99 | 64 | 0.0284 | 0.0062 | 0.02222 | 0.0486 | 78.315 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 0.1126 | 0.0894 | 0.02320 | 0.0255 | 20.599 |

FORECASTS OF REPLENISHMENT FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.1185 | 0.1144 | 0.00409 | 0.0908 | 3.450 |
| 2 | 0 | 4 | 64 | 0.0919 | 0.0841 | 0.00784 | 0.0654 | 8.525 |
| 2 | 0 | 99 | 32 | 0.1135 | 0.1099 | 0.00356 | 0.0796 | 3.136 |
| 2 | 0 | 99 | 64 | 0.0915 | 0.0826 | 0.00889 | 0.0736 | 9.716 |
| 2 | 4 | 4 | 32 | 0.0977 | 0.0919 | 0.00577 | 0.0769 | 5.906 |
| 2 | 4 | 4 | 64 | 0.0808 | 0.0783 | 0.00250 | 0.0609 | 3.095 |
| 2 | 4 | 99 | 32 | 0.0819 | 0.0885 | -0.00654 | 0.0645 | 7.981 |
| 2 | 4 | 99 | 64 | 0.0635 | 0.0622 | 0.00130 | 0.0499 | 2.045 |
| 8 | 0 | 4 | 32 | 0.2623 | 0.2557 | 0.00659 | 0.0979 | 2.511 |
| 8 | 0 | 4 | 64 | 0.2008 | 0.1921 | 0.00870 | 0.0768 | 4.334 |
| 8 | 0 | 99 | 32 | 0.2527 | 0.2505 | 0.00221 | 0.0905 | 0.875 |
| 8 | 0 | 99 | 64 | 0.1942 | 0.1899 | 0.00428 | 0.0709 | 2.203 |
| 8 | 4 | 4 | 32 | 0.2354 | 0.2260 | 0.00937 | 0.0922 | 3.983 |
| 8 | 4 | 4 | 64 | 0.1777 | 0.1736 | 0.00413 | 0.0667 | 2.327 |
| 8 | 4 | 99 | 32 | 0.2054 | 0.2024 | 0.00303 | 0.0870 | 1.475 |
| 8 | 4 | 99 | 64 | 0.1635 | 0.1547 | 0.00880 | 0.0732 | 5.382 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 0.1519 | 0.1473 | 0.00466 | 0.0193 | 3.065 |

TABLE E3: SINGLE ITEM FORECASTS OF TOTAL COST USING FUTURE DEMAND GENERATION (NEGATIVE BINOMIAL)

FORECASTS OF COST PER PERIOD IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 12.5569 | 11.9856 | 0.57130 | 4.6172 | 4.550 |
| 2 | 0 | 4 | 64 | 15.7999 | 14.7195 | 1.08045 | 5.8695 | 6.838 |
| 2 | 0 | 99 | 32 | 37.3637 | 26.6897 | 10.67433 | 19.2508 | 28.569 |
| 2 | 0 | 99 | 64 | 40.9674 | 28.2263 | 12.74139 | 20.4562 | 31.101 |
| 2 | 4 | 4 | 32 | 22.3079 | 17.3179 | 4.99010 | 7.3599 | 22.369 |
| 2 | 4 | 4 | 64 | 25.4481 | 21.5633 | 3.88505 | 8.2790 | 15.267 |
| 2 | 4 | 99 | 32 | 68.0734 | 44.8661 | 23.20738 | 46.7514 | 34.092 |
| 2 | 4 | 99 | 64 | 72.1237 | 42.9976 | 29.12570 | 44.0360 | 40.383 |
| 8 | 0 | 4 | 32 | 26.1009 | 25.2297 | 0.87119 | 6.6037 | 3.338 |
| 8 | 0 | 4 | 64 | 32.9436 | 32.0066 | 0.93705 | 7.4942 | 2.844 |
| 8 | 0 | 99 | 32 | 59.1978 | 48.6003 | 10.59739 | 24.7636 | 17.902 |
| 8 | 0 | 99 | 64 | 66.3079 | 54.8398 | 11.46825 | 25.6898 | 17.295 |
| 8 | 4 | 4 | 32 | 45.7158 | 38.2902 | 7.42567 | 11.9220 | 16.243 |
| 8 | 4 | 4 | 64 | 49.7073 | 43.3556 | 6.35165 | 11.5746 | 12.778 |
| 8 | 4 | 99 | 32 | 112.1910 | 81.5998 | 30.59058 | 56.1924 | 27.267 |
| 8 | 4 | 99 | 64 | 119.5264 | 84.8407 | 34.68488 | 69.1812 | 29.019 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 806.3311 | 617.1274 | 189.20229 | 121.1508 | 23.465 |

## APPENDIX F

TABLE F1: SINGLE ITEM FORECASTS OF HOLDING AND BACKLOG QUANTITY USING FUTURE DEMAND GENERATION WITH VARIABLE MEANS AND VARIANCES

FORECASTS CF HOLDING QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FOBCASTS | DISPERSION CF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 5.6002 | 5.5730 | 0.02724 | 2.1835 | 0.486 |
| 2 | 0 | 4 | 64 | 6.4698 | 6.4620 | 0.00779 | 2.7491 | 0.120 |
| 2 | 0 | 99 | 32 | 17.1250 | 17.2877 | -0.16272 | 2.9415 | 0.950 |
| 2 | 0 | 99 | 64 | 17.2720 | 18.0381 | -0.76617 | 3.6471 | 4.436 |
| 2 | 4 | 4 | 32 | 10.2225 | 10.7114 | -0.48893 | 5.4090 | 4.783 |
| 2 | 4 | 4 | 64 | 12.1502 | 12.0587 | 0.09150 | 6.5922 | 0.753 |
| 2 | 4 | 99 | 32 | 38.7644 | 40.2291 | -1.46485 | 9.5856 | 3.779 |
| 2 | 4 | 99 | 64 | 34.9989 | 37.1003 | -2.10157 | 8.5277 | 6.005 |
| 8 | 0 | 4 | 32 | 11.2149 | 10.8108 | 0.40408 | 2.2844 | 3.603 |
| 8 | 0 | 4 | 64 | 12.8777 | 12.6509 | 0.22678 | 2.6476 | 1.761 |
| 8 | 0 | 99 | 32 | 31.5095 | 31.9251 | -0.41563 | 3.5126 | 1.319 |
| 8 | 0 | 99 | 64 | 33.2635 | 33.8071 | -0.54368 | 4.0642 | 1.634 |
| 8 | 4 | 4 | 32 | 23.7531 | 22.7546 | 0.99864 | 11.2741 | 4.204 |
| 8 | 4 | 4 | 64 | 22.1750 | 22.1771 | -0.00211 | 9.5267 | 0.010 |
| 8 | 4 | 99 | 32 | 70.7571 | 71.8123 | -1.05524 | 14.7176 | 1.491 |
| 8 | 4 | 99 | 64 | 68.6904 | 69.9754 | -1.28484 | 14.8564 | 1.870 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 416.8440 | 423.3733 | -6.52970 | 31.1105 | 1.566 |

FORECASTS OF BACKLOG QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.7915 | 0.6612 | 0.13022 | 0.5827 | 16.453 |
| 2 | 0 | 4 | 64 | 0.8618 | 0.7068 | 0.15500 | 0.6817 | 17.986 |
| 2 | 0 | 99 | 32 | 0.1678 | 0.0537 | 0.11406 | 0.1828 | 67.990 |
| 2 | 0 | 99 | 64 | 0.1802 | 0.0401 | 0.14008 | 0.1696 | 77.747 |
| 2 | 4 | 4 | 32 | 2.2398 | 1.2320 | 1.00789 | 2.0103 | 44.999 |
| 2 | 4 | 4 | 64 | 2.0322 | 1.5546 | 0.47765 | 2.3146 | 23.504 |
| 2 | 4 | 99 | 32 | 0.2696 | 0.0383 | 0.23130 | 0.5270 | 85.804 |
| 2 | 4 | 99 | 64 | 0.3340 | 0.0311 | 0.30287 | 0.4800 | 90.686 |
| 8 | 0 | 4 | 32 | 1.6231 | 1.5445 | 0.07855 | 1.1286 | 4.840 |
| 8 | 0 | 4 | 64 | 1.8043 | 1.8839 | -0.07962 | 1.2703 | 4.413 |
| 8 | 0 | 99 | 32 | 0.1980 | 0.1022 | 0.09579 | 0.2685 | 48.379 |
| 8 | 0 | 99 | 64 | 0.2082 | 0.1015 | 0.10668 | 0.2607 | 51.236 |
| 8 | 4 | 4 | 32 | 3.6077 | 3.2352 | 0.37245 | 4.5580 | 10.324 |
| 8 | 4 | 4 | 64 | 4.0402 | 3.5259 | 0.51424 | 4.2948 | 12.728 |
| 8 | 4 | 99 | 32 | 0.3521 | 0.0476 | 0.30449 | 0.6383 | 86.470 |
| 8 | 4 | 99 | 64 | 0.4078 | 0.0889 | 0.31886 | 0.7578 | 78.191 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 19.1182 | 14.8477 | 4.27053 | 7.3486 | 22.337 |

TABLE F2: SINGLE ITEM FORECASTS OF BACKLOG AND REPLENISHMENT FREQUENCY USING FUTURE DEMAND GENERATION WITH VARIABLE MEANS AND VARIANCES

FORECASTS OF BACKLOG FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.1494 | 0.1511 | -0.00178 | 0.0970 | 1.192 |
| 2 | 0 | 4 | 64 | 0.1794 | 0.1733 | 0.00614 | 0.1476 | 3.420 |
| 2 | 0 | 99 | 32 | 0.0239 | 0.0094 | 0.01446 | 0.0259 | 60.540 |
| 2 | 0 | 99 | 64 | 0.0259 | 0.0084 | 0.01753 | 0.0252 | 67.567 |
| 2 | 4 | 4 | 32 | 0.2288 | 0.1631 | 0.06569 | 0.1655 | 28.711 |
| 2 | 4 | 4 | 64 | 0.2073 | 0.1818 | 0.02552 | 0.1767 | 12.311 |
| 2 | 4 | 99 | 32 | 0.0283 | 0.0049 | 0.02340 | 0.0525 | 82.818 |
| 2 | 4 | 99 | 64 | 0.0351 | 0.0058 | 0.02925 | 0.0492 | 83.412 |
| 8 | 0 | 4 | 32 | 0.1803 | 0.1805 | -0.00013 | 0.0771 | 0.074 |
| 8 | 0 | 4 | 64 | 0.2000 | 0.2052 | -0.00522 | 0.0877 | 2.610 |
| 8 | 0 | 99 | 32 | 0.0220 | 0.0114 | 0.01056 | 0.0260 | 47.990 |
| 8 | 0 | 99 | 64 | 0.0231 | 0.0107 | 0.01241 | 0.0237 | 53.660 |
| 8 | 4 | 4 | 32 | 0.2112 | 0.1996 | 0.01160 | 0.1895 | 5.494 |
| 8 | 4 | 4 | 64 | 0.2341 | 0.2145 | 0.01958 | 0.1674 | 8.365 |
| 8 | 4 | 99 | 32 | 0.0249 | 0.0036 | 0.02126 | 0.0410 | 85.497 |
| 8 | 4 | 99 | 64 | 0.0284 | 0.0075 | 0.02088 | 0.0481 | 73.571 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 0.1126 | 0.0957 | 0.01695 | 0.0265 | 15.047 |

FORECASTS OF REPLENISHMENT FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.1185 | 0.1193 | -0.00082 | 0.0886 | 0.690 |
| 2 | 0 | 4 | 64 | 0.0919 | 0.0854 | 0.00649 | 0.0661 | 7.061 |
| 2 | 0 | 99 | 32 | 0.1135 | 0.1155 | -0.00207 | 0.0827 | 1.822 |
| 2 | 0 | 99 | 64 | 0.0915 | 0.0844 | 0.00716 | 0.0736 | 7.826 |
| 2 | 4 | 4 | 32 | 0.0977 | 0.0955 | 0.00216 | 0.0758 | 2.215 |
| 2 | 4 | 4 | 64 | 0.0808 | 0.0800 | 0.00077 | 0.0602 | 0.952 |
| 2 | 4 | 99 | 32 | 0.0819 | 0.0892 | -0.00731 | 0.0629 | 8.920 |
| 2 | 4 | 99 | 64 | 0.0635 | 0.0652 | -0.00173 | 0.0515 | 2.727 |
| 8 | 0 | 4 | 32 | 0.2623 | 0.2556 | 0.00668 | 0.0991 | 2.548 |
| 8 | 0 | 4 | 64 | 0.2008 | 0.1926 | 0.00817 | 0.0788 | 4.071 |
| 8 | 0 | 99 | 32 | 0.2527 | 0.2512 | 0.00149 | 0.0945 | 0.590 |
| 8 | 0 | 99 | 64 | 0.1942 | 0.1923 | 0.00197 | 0.0711 | 1.015 |
| 8 | 4 | 4 | 32 | 0.2354 | 0.2251 | 0.01029 | 0.0956 | 4.371 |
| 8 | 4 | 4 | 64 | 0.1777 | 0.1702 | 0.00745 | 0.0701 | 4.194 |
| 8 | 4 | 99 | 32 | 0.2054 | 0.2027 | 0.00269 | 0.0862 | 1.311 |
| 8 | 4 | 99 | 64 | 0.1635 | 0.1540 | 0.00947 | 0.0751 | 5.794 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 0.1519 | 0.1486 | 0.00331 | 0.0195 | 2.175 |

TABLE F3: SINGLE ITEM FORECASTS OF TOTAL COST USING FUTURE DEMAND GENERATION WITH VARIABLE MEANS AND VARIANCES

FORECASTS OF COST PER PERIOD IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 12.5569 | 12.0349 | 0.52198 | 4.3522 | 4.157 |
| 2 | 0 | 4 | 64 | 15.7999 | 14.7567 | 1.04319 | 6.0107 | 6.602 |
| 2 | 0 | 99 | 32 | 37.3637 | 26.3007 | 11.06342 | 18.1632 | 29.610 |
| 2 | 0 | 99 | 64 | 40.9674 | 27.4072 | 13.56062 | 18.6490 | 33.101 |
| 2 | 4 | 4 | 32 | 22.3079 | 18.6962 | 3.61180 | 7.4276 | 16.191 |
| 2 | 4 | 4 | 64 | 25.4481 | 23.3969 | 2.05129 | 8.6748 | 8.061 |
| 2 | 4 | 99 | 32 | 68.0734 | 46.8729 | 21.20052 | 49.1702 | 31.144 |
| 2 | 4 | 99 | 64 | 72.1237 | 44.3519 | 27.77144 | 45.5972 | 38.505 |
| 8 | 0 | 4 | 32 | 26.1009 | 25.1687 | 0.93215 | 5.7572 | 3.571 |
| 8 | 0 | 4 | 64 | 32.9436 | 32.5123 | 0.43138 | 7.8071 | 1.309 |
| 8 | 0 | 99 | 32 | 59.1978 | 50.0820 | 9.11561 | 26.7811 | 15.399 |
| 8 | 0 | 99 | 64 | 66.3079 | 56.1636 | 10.14436 | 26.1362 | 15.299 |
| 8 | 4 | 4 | 32 | 45.7158 | 42.8982 | 2.81763 | 13.8725 | 6.163 |
| 8 | 4 | 4 | 64 | 49.7073 | 47.1755 | 2.53175 | 13.4428 | 5.093 |
| 8 | 4 | 99 | 32 | 112.1910 | 83.0152 | 29.17517 | 57.7132 | 26.005 |
| 8 | 4 | 99 | 64 | 119.5264 | 88.6358 | 30.88965 | 70.1502 | 25.843 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 806.3311 | 639.4680 | 166.86191 | 124.4163 | 20.694 |

## APPENDIX G

TABLE G1: SINGLE ITEM FORECASTS OF HOLDING AND BACKLOG QUANTITY USING FUTURE DEMAND GENERATION (GAMMA)

FORECASTS OF HOLDING QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 5.6002 | 5.2350 | 0.36519 | 2.0269 | 6.521 |
| 2 | 0 | 4 | 64 | 6.4698 | 6.1239 | 0.34584 | 2.5339 | 5.346 |
| 2 | 0 | 99 | 32 | 17.1250 | 16.6912 | 0.43379 | 2.6750 | 2.533 |
| 2 | 0 | 99 | 64 | 17.2720 | 17.4005 | -0.12849 | 3.5594 | 0.744 |
| 2 | 4 | 4 | 32 | 10.2225 | 9.7260 | 0.49645 | 5.0951 | 4.857 |
| 2 | 4 | 4 | 64 | 12.1502 | 10.9911 | 1.15912 | 5.4447 | 9.540 |
| 2 | 4 | 99 | 32 | 38.7644 | 38.9375 | -0.17323 | 8.7272 | 0.447 |
| 2 | 4 | 99 | 64 | 34.9989 | 35.1542 | -0.15524 | 8.6762 | 0.444 |
| 8 | 0 | 4 | 32 | 11.2149 | 10.3333 | 0.88162 | 2.5777 | 7.861 |
| 8 | 0 | 4 | 64 | 12.8777 | 11.8272 | 1.05050 | 2.8801 | 8.157 |
| 8 | 0 | 99 | 32 | 31.5095 | 30.8098 | 0.69983 | 3.7208 | 2.221 |
| 8 | 0 | 99 | 64 | 33.2635 | 32.8495 | 0.41407 | 4.0390 | 1.245 |
| 8 | 4 | 4 | 32 | 23.7531 | 19.2636 | 4.48965 | 13.3827 | 18.901 |
| 8 | 4 | 4 | 64 | 22.1750 | 19.4881 | 2.68696 | 12.0868 | 12.117 |
| 8 | 4 | 99 | 32 | 70.7571 | 68.3355 | 2.42164 | 18.1044 | 3.422 |
| 8 | 4 | 99 | 64 | 68.6904 | 66.2595 | 2.43110 | 17.0393 | 3.539 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 416.8440 | 399.4255 | 17.41884 | 35.0113 | 4.179 |

FORECASTS OF BACKLOG QUANTITY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.7915 | 0.7534 | 0.03809 | 0.5441 | 4.812 |
| 2 | 0 | 4 | 64 | 0.8618 | 0.9023 | -0.04048 | 0.8028 | 4.697 |
| 2 | 0 | 99 | 32 | 0.1678 | 0.0863 | 0.08148 | 0.2081 | 48.571 |
| 2 | 0 | 99 | 64 | 0.1802 | 0.0779 | 0.10229 | 0.1989 | 56.771 |
| 2 | 4 | 4 | 32 | 2.2398 | 1.2357 | 1.00412 | 2.0353 | 44.830 |
| 2 | 4 | 4 | 64 | 2.0322 | 1.6514 | 0.38079 | 2.4904 | 18.738 |
| 2 | 4 | 99 | 32 | 0.2696 | 0.0831 | 0.18646 | 0.6021 | 69.168 |
| 2 | 4 | 99 | 64 | 0.3340 | 0.1949 | 0.13910 | 0.9114 | 41.649 |
| 8 | 0 | 4 | 32 | 1.6231 | 1.8031 | -0.18000 | 1.0912 | 11.090 |
| 8 | 0 | 4 | 64 | 1.8043 | 2.1655 | -0.36122 | 1.3077 | 20.019 |
| 8 | 0 | 99 | 32 | 0.1980 | 0.1272 | 0.07076 | 0.2271 | 35.739 |
| 8 | 0 | 99 | 64 | 0.2082 | 0.1437 | 0.06450 | 0.2572 | 30.974 |
| 8 | 4 | 4 | 32 | 3.6077 | 5.2336 | -1.62587 | 5.4759 | 45.066 |
| 8 | 4 | 4 | 64 | 4.0402 | 5.1108 | -1.07063 | 4.9899 | 26.500 |
| 8 | 4 | 99 | 32 | 0.3521 | 0.2098 | 0.14230 | 0.6620 | 40.411 |
| 8 | 4 | 99 | 64 | 0.4078 | 0.3585 | 0.04937 | 1.3380 | 12.105 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 19.1182 | 20.1372 | -1.01895 | 8.5261 | 5.330 |

TABLE G2: SINGLE ITEM FORECASTS OF BACKLOG AND REPLENISHMENT FREQUENCY USING FUTURE DEMAND GENERATION (GAMMA)

FORECASTS OF BACKLOG FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.1494 | 0.2069 | -0.05751 | 0.0996 | 38.502 |
| 2 | 0 | 4 | 64 | 0.1794 | 0.2379 | -0.05843 | 0.1581 | 32.560 |
| 2 | 0 | 99 | 32 | 0.0239 | 0.0152 | 0.00869 | 0.0284 | 36.382 |
| 2 | 0 | 99 | 64 | 0.0259 | 0.0152 | 0.01071 | 0.0256 | 41.288 |
| 2 | 4 | 4 | 32 | 0.2288 | 0.1786 | 0.05020 | 0.1839 | 21.941 |
| 2 | 4 | 4 | 64 | 0.2073 | 0.1978 | 0.00945 | 0.1832 | 4.560 |
| 2 | 4 | 99 | 32 | 0.0283 | 0.0127 | 0.01557 | 0.0650 | 55.088 |
| 2 | 4 | 99 | 64 | 0.0351 | 0.0250 | 0.01003 | 0.0953 | 28.605 |
| 8 | 0 | 4 | 32 | 0.1803 | 0.2189 | -0.03858 | 0.0995 | 21.391 |
| 8 | 0 | 4 | 64 | 0.2000 | 0.2495 | -0.04953 | 0.1063 | 24.768 |
| 8 | 0 | 99 | 32 | 0.0220 | 0.0187 | 0.00331 | 0.0208 | 15.037 |
| 8 | 0 | 99 | 64 | 0.0231 | 0.0173 | 0.00579 | 0.0212 | 25.025 |
| 8 | 4 | 4 | 32 | 0.2112 | 0.3009 | -0.08972 | 0.2249 | 42.486 |
| 8 | 4 | 4 | 64 | 0.2341 | 0.2931 | -0.05901 | 0.2055 | 25.209 |
| 8 | 4 | 99 | 32 | 0.0249 | 0.0212 | 0.00367 | 0.0487 | 14.762 |
| 8 | 4 | 99 | 64 | 0.0284 | 0.0299 | -0.00151 | 0.0788 | 5.309 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 0.1126 | 0.1274 | -0.01480 | 0.0307 | 13.144 |

FORECASTS OF REPLENISHMENT FREQUENCY IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 0.1185 | 0.1155 | 0.00300 | 0.0880 | 2.532 |
| 2 | 0 | 4 | 64 | 0.0919 | 0.0983 | -0.00642 | 0.0830 | 6.987 |
| 2 | 0 | 99 | 32 | 0.1135 | 0.1117 | 0.00177 | 0.0855 | 1.559 |
| 2 | 0 | 99 | 64 | 0.0915 | 0.0869 | 0.00465 | 0.0732 | 5.084 |
| 2 | 4 | 4 | 32 | 0.0977 | 0.0948 | 0.00285 | 0.0781 | 2.913 |
| 2 | 4 | 4 | 64 | 0.0808 | 0.0777 | 0.00304 | 0.0613 | 3.762 |
| 2 | 4 | 99 | 32 | 0.0819 | 0.0876 | -0.00565 | 0.0691 | 6.901 |
| 2 | 4 | 99 | 64 | 0.0635 | 0.0664 | -0.00296 | 0.0574 | 4.667 |
| 8 | 0 | 4 | 32 | 0.2623 | 0.2701 | -0.00781 | 0.1165 | 2.976 |
| 8 | 0 | 4 | 64 | 0.2008 | 0.2103 | -0.00954 | 0.0921 | 4.751 |
| 8 | 0 | 99 | 32 | 0.2527 | 0.2670 | -0.01427 | 0.1111 | 5.647 |
| 8 | 0 | 99 | 64 | 0.1942 | 0.2033 | -0.00904 | 0.0790 | 4.653 |
| 8 | 4 | 4 | 32 | 0.2354 | 0.2478 | -0.01242 | 0.1082 | 5.278 |
| 8 | 4 | 4 | 64 | 0.1777 | 0.1823 | -0.00462 | 0.0810 | 2.597 |
| 8 | 4 | 99 | 32 | 0.2054 | 0.2139 | -0.00854 | 0.1014 | 4.157 |
| 8 | 4 | 99 | 64 | 0.1635 | 0.1658 | -0.00235 | 0.0837 | 1.435 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 0.1519 | 0.1562 | -0.00427 | 0.0218 | 2.810 |

TABLE G3: SINGLE ITEM FORECASTS OF TOTAL COST USING FUTURE DEMAND GENERATION (GAMMA)

FORECASTS OF COST PER PERIOD IN A 16-ITEM SYSTEM

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMANDS: VARIANCE TO MEAN RATIO = 9.0 REVISION INTERVAL = 13

| MEAN | LEADTIME | ITEMS: C(OUT)/C(IN) | C(FIX)/C(IN) | AVERAGE ACTUAL VALUE | AVERAGE FORECAST VALUE | ACTUAL VALUES LESS FORCASTS | DISPERSION OF BIAS | PERCENTAGE ERROR |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 4 | 32 | 12.5569 | 11.9433 | 0.61354 | 4.2281 | 4.886 |
| 2 | 0 | 4 | 64 | 15.7999 | 16.0271 | -0.22714 | 7.3254 | 1.438 |
| 2 | 0 | 99 | 32 | 37.3637 | 28.8066 | 8.55736 | 20.7087 | 22.903 |
| 2 | 0 | 99 | 64 | 40.9674 | 30.6716 | 10.29603 | 20.7147 | 25.132 |
| 2 | 4 | 4 | 32 | 22.3079 | 17.7040 | 4.60394 | 7.6644 | 20.638 |
| 2 | 4 | 4 | 64 | 25.4481 | 22.5715 | 2.87673 | 9.9720 | 11.304 |
| 2 | 4 | 99 | 32 | 68.0734 | 49.9683 | 18.10547 | 57.5825 | 26.597 |
| 2 | 4 | 99 | 64 | 72.1237 | 58.6978 | 13.42587 | 88.1035 | 18.615 |
| 8 | 0 | 4 | 32 | 26.1009 | 26.1890 | -0.08823 | 5.6886 | 0.338 |
| 8 | 0 | 4 | 64 | 32.9436 | 33.9485 | -1.00482 | 8.6387 | 3.050 |
| 8 | 0 | 99 | 32 | 59.1978 | 51.9490 | 7.24885 | 22.2945 | 12.245 |
| 8 | 0 | 99 | 64 | 66.3079 | 60.0873 | 6.22062 | 25.6957 | 9.381 |
| 8 | 4 | 4 | 32 | 45.7158 | 48.1272 | -2.41131 | 16.6180 | 5.275 |
| 8 | 4 | 4 | 64 | 49.7073 | 51.5983 | -1.89082 | 15.9227 | 3.804 |
| 8 | 4 | 99 | 32 | 112.1910 | 95.9547 | 16.23627 | 61.7128 | 14.472 |
| 8 | 4 | 99 | 64 | 119.5264 | 112.3581 | 7.16835 | 127.6945 | 5.997 |
| TOTALS FOR THE ENTIRE 16-ITEM SYSTEM : | | | | 806.3311 | 716.6011 | 89.73062 | 184.7298 | 11.128 |